Global Optimality Integral Conditions and an Algorithm for Multiobjective Problems

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Outline

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Motivation

Differential Characterization

- Local
- Requires several degrees of differentiability
- Convexity or generalized convexity

Integral Characterization

- Global
- It does not even require continuity

Outline

Scalar Optimization

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & x \in X \subseteq \mathbb{R}^n \end{array} \qquad \text{(P)}$$

- $f : \mathbb{R}^n \to \mathbb{R}$
- $X \subseteq \mathbb{R}^n$ is nonempty

Global Optimality Integral Conditions

Proposed originally by Falk (1973) for maximization problems

- $X \subset \mathbb{R}^n$ is a compact set with nonempty interior
- $f : X \to (-\infty, 0)$ is a continuous function
- $\overline{x} \in X$ such that $f(\overline{x}) = -1$.

Theorem

The integral $\Upsilon(t) = \int_X [-f(x)]^t \, dx$ converges, when $t \to \infty$ if, and only if, $\overline{x}$ is a global solution of the problem (P).

- The hypothesis $f < 0$ is not very restrictive, as the function $x \mapsto -\exp(-f(x))$ is negative and its minimizers coincide with the minimizers of f.

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Corollary:

Let the sequence $\{\Upsilon_n\}_{n\in\mathbb{N}}$, be given by $\Upsilon_n = \int_X [-f(x)]^n dx$. Then:

(i) $\overline{x} \in X$ is a global optimal solution of (P) if and only if $\{\Upsilon_n\}_{n\in\mathbb{N}}$ converges.

(ii) If for some n, we have $\Upsilon_{n+1} > \Upsilon_n$, then $\overline{x}$ is not a global optimal solution of problem (P).

May be useful in eliminating $\overline{x}$ candidates that are not global minimizers.

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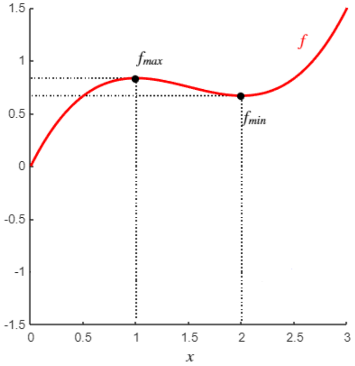
$$\text{minimize } f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x$$
$$\text{subject to } x \in X = [0, 3]$$



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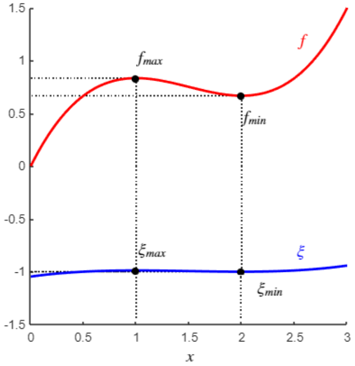

- $\xi(x) = \frac{3}{43}\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x - 15\right)$
- f and ξ have the same local and global minimizers.
- $\Upsilon_{20} = \int_0^3 [-\xi(x)]^{20} dx \approx 2.8226,$
- $\Upsilon_{21} = \int_0^3 [-\xi(x)]^{21} dx \approx 2.8230.$
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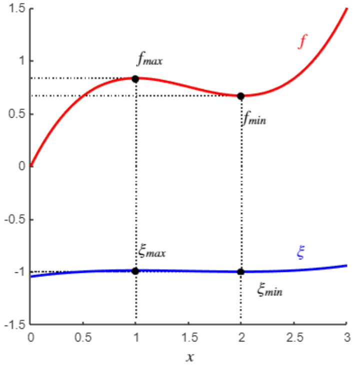

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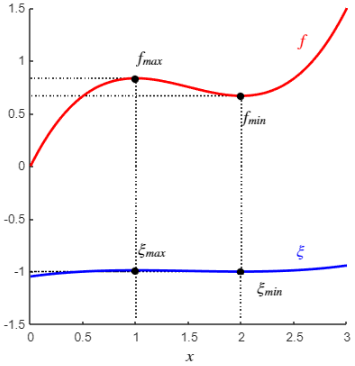

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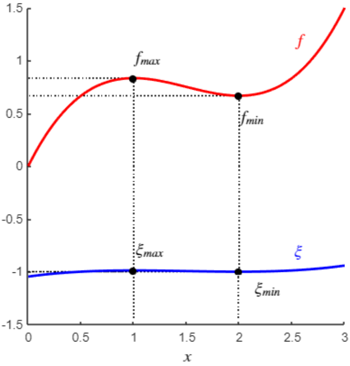

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- For each $c \in \mathbb{R}$, we can define $H_c = \{x \in \mathbb{R}^n \mid f(x) \leq c\}$.

A1. X is robust, i.e.,

$$cl(X) = cl\ (int\ X).$$

A2. The function $f : X \to \mathbb{R}$ is lower semicontinuous and upper robust

A function $f : \mathbb{R}^n \to \mathbb{R}$ is upper robust over X if, and only if, the set $\{x \in X \mid f(x) < c\}$ is robust, for each $c \in \mathbb{R}$.

A3. There exists $c \in \mathbb{R}$ such that $H_c \cap X$ is a compact set.

Theorem

If $\mu(H_{\bar{c}} \cap X) = 0$, then $\bar{c}$ is the optimum value of f over X and $H_{\bar{c}} \cap X$ is the set of global minimizers.

Remark: μ denotes the Lebesgue measure in $\mathbb{R}^n$

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Case $c > \overline{c}$

Let $\overline{c}$ be the minimum value of f over X. For $c > \overline{c}$

$$M(f, c, X) = \frac{1}{\mu(H_c \cap X)} \int_{H_c \cap X} f(x) d\mu.$$

$$V(f, c, X) = \frac{1}{\mu(H_c \cap X)} \int_{H_c \cap X} [f(x) - M(f, c)]^2 d\mu$$

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Let $\{c_k\}_{k\in\mathbb{N}}$ be a decreasing sequence converging to c.

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A2. The function $f : X \to \mathbb{R}$ is lower semicontinuous and upper robust

A3. There exists $c \in \mathbb{R}$ such that $H_c \cap X$ is a compact set.

Theorem (Thm. 5.1 by Zheng, (1991))

Suppose that Assumptions **A1**, **A2**, and **A3** hold. The following statements are equivalent:

(i) $\bar{x} \in X$ is a global minimizer of (P) and $\bar{c} = f(\bar{x})$ is the global minimum value of f over X

(ii) $M(f, \bar{c}, X) = \bar{c}$

(iii) $V(f, \bar{c}, X) = 0$

(iv) $V_1(f, \bar{c}, X) = 0$

Example

Fixed $a > 0$,

$$\begin{aligned} &\text{minimize } f(x) = |x|^a \\ &x \in \mathbb{R} \end{aligned}$$

- $H_c = [-c^{1/a}, c^{1/a}]$
- $M(f, c, \mathbb{R}) = \dfrac{1}{2c^{1/a}} \displaystyle\int_{-c^{1/a}}^{c^{1/a}} |x|^a dx = \dfrac{1}{1+a} c$
- $V_1(f, c, \mathbb{R}) = \dfrac{1}{2c^{1/a}} \displaystyle\int_{-c^{1/a}}^{c^{1/a}} (|x|^a - c)^2 dx = \dfrac{2a^2}{(1+a)(1+2a)} c^2$

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- $f_\ell : \mathbb{R}^n \to \mathbb{R},\ \ell = 1, 2, \ldots, r \geq 2$
- $X \subseteq \mathbb{R}^n$ is nonempty
- objectives often conflicting

Scalarization Techniques

Let $W = \{w \in \mathbb{R}^r \mid w_\ell \geq 0,\ \forall \ell \in \{1, \ldots, r\}\ \wedge\ \|w\|_1 = 1\}$ and $W^* = \{w \in \mathbb{R}^r \mid w_\ell > 0,\ \forall \ell \in \{1, \ldots, r\}\ \wedge\ \|w\|_1 = 1\}$

Weighted sum

$$\begin{aligned} &\text{Minimize} \ \sum_{\ell=1}^{r} w_\ell f_\ell(x) = \Phi_w(x) \\ &\text{subject to} \ \ x \in X \end{aligned} \tag{WS}$$

Theorem (Thm. 3.1.1 and 3.1.2 - Miettinen, 1999)

If there exists $w \in W$ (respectively, $w \in W^*$) such that $\overline{x} \in X$ is a solution of (WS) then $\overline{x}$ is a weak Pareto optimal solution (respectively, Pareto optimal solution) of (MOP).

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Scalarization Techniques

Consider the utopian objective vector $u^* = f^* - \xi$, where $f^*_\ell = \min_{x \in X} f_\ell(x)$ for $\ell \in \{1, \ldots, r\}$ and $\xi \in \mathbb{R}^r_+$

Chebyshev weighted

$$\begin{aligned} &\text{Minimize } ||f(x) - u^*||^w_\infty = \Psi_w(x) \\ &\text{subject to } \ x \in X \end{aligned} \qquad (\text{WCS}_w)$$

where $||f(x) - u^*||^w_\infty = \max_\ell \{w_\ell(f_\ell(x) - u^*_\ell)\}$.

Theorem (Thm. 3.4.2 and 3.4.5 - Miettinen, 1999)

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Consider the utopian objective vector $u^* = f^* - \xi$, where $f^*_\ell = \min_{x \in X} f_\ell(x)$ for $\ell \in \{1, \dots, r\}$ and $\xi \in \mathbb{R}^r_+$

Chebyshev weighted

$$\begin{aligned} &\text{Minimize } \; ||f(x) - u^*||^w_\infty = \Psi_w(x) \\ &\text{subject to } \; x \in X \end{aligned} \qquad (\mathsf{WCS}_w)$$

where $||f(x) - u^*||^w_\infty = \max_\ell \{w_\ell(f_\ell(x) - u^*_\ell)\}$.

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Consider that f_ℓ, for $\ell = 1, \ldots, r$, are continuous functions and X is a compact set.

$\Phi_w(x) = \sum_{\ell=1}^{r} w_\ell f_\ell(x)$ and $\Psi_w(x) = ||f(x) - u^*||_\infty^w$ have no image in the interval $(-\infty, 0]$

- Φ_w and Ψ_w are continuous functions into the compact set X.

$$\begin{array}{lcl} \Phi_w(x) \leq M_1 & \Rightarrow & \widetilde{\Phi}_w = \Phi_w - M_1 \leq 0 \\ \Psi_w(x) \leq M_2 & \Rightarrow & \widetilde{\Psi}_w = \Psi_w - M_2 \leq 0 \end{array}$$

$\bar{x} \in X$ is a global minimizer of Φ_w over $X \iff \bar{x}$ minimizes the function $x \mapsto -\frac{\widetilde{\Phi}_w(x)}{\widetilde{\Phi}_w(\bar{x})}$ on X. A similar result holds for the function Ψ_w.

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Theorem

Consider $\overline{x} \in X$ and $w \in W$ (respectively, $w \in W^*$). If $\Upsilon_w(t) = \int_X \left[\frac{\widetilde{\Phi}_w(x)}{\widetilde{\Phi}_w(\overline{x})} \right]^t d\mu$ converges as $t \to \infty$, then $\overline{x}$ is a weak Pareto optimal solution (respectively, Pareto optimal solution) for the problem (MOP).

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A point $\overline{x} \in X$ is a weak Pareto optimal solution of (MOP) if, and only if, there exists $w \in W^*$ such that the function defined by $\Upsilon_w(t) = \int_X \left[\frac{\widetilde{\Psi}_w(x)}{\widetilde{\Psi}_w(\overline{x})} \right]^t d\mu$ converges when $t \to \infty$.

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A1. X is robust.

A2. The function $f : X \to \mathbb{R}$ is lower semicontinuous and upper robust.

A3. There exists $c \in \mathbb{R}$ such that $H_c \cap X$ is a compact set.

Theorem

Let Assumption **A1** holds. Suppose that there exists $w \in W$ (respectively, $w \in W^*$) such that the function Φ_w satisfies Assumptions **A2** and **A3**. Consider $\overline{x} \in X$ and $\overline{c} = \Phi_w(\overline{x})$. Then the following conditions are equivalent:

(i) $\overline{x} \in X$ is a solution of the problem (WS)

(ii) $M(\Phi_w, \overline{c}, X) = \overline{c}$

(iii) $V(\Phi_w, \overline{c}, X) = 0$

(iv) $V_1(\Phi_w, \overline{c}, X) = 0$

where M, V, and V_1 are, respectively, the mean value, the variance and the modified variance of Φ_w. Moreover, in these equivalent situations, $\overline{x}$ is a weak Pareto optimal solution (respectively, Pareto optimal solution) of (MOP).

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A2. The function $f : X \to \mathbb{R}$ is lower semicontinuous and upper robust.

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A1′. X is a robust and closed set.

A2′. The functions f_ℓ, $\ell = 1, \cdots, r$, are continuous.

A3′. There exist an index ℓ_0 and $c_0 \in \mathbb{R}$ such that the set $\{x \in X \mid f_{\ell_0}(x) \le c_0\}$ is compact.

Proposition

Suppose that **A1′**, **A2′**, and **A3′** hold. Then, **A1** holds and for all $w \in W^*$, the functions Φ_w and Ψ_w satisfy Assumptions **A2** and **A3**.

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Necessary and Sufficient Conditions

Theorem

Suppose that the problem (MOP) satisfies **A1′**, **A2′**, and **A3′**. Consider $\overline{x} \in X$. Then, the following conditions are equivalent:

(i) $\overline{x}$ is a weak Pareto optimal solution of (MOP)

(ii) there exists $w \in W^*$ such that $\overline{x}$ minimizes Ψ_w over X and $\overline{c} = \Psi_w(\overline{x})$

(iii) there exists $w \in W^*$ such that $M(\Psi_w, \overline{c}, X) = \overline{c}$, with $\overline{c} = \Psi_w(\overline{x})$

(iv) there exists $w \in W^*$ such that $V(\Psi_w, \overline{c}, X) = 0$, with $\overline{c} = \Psi_w(\overline{x})$

(v) there exists $w \in W^*$ such that $V_1(\Psi_w, \overline{c}, X) = 0$, with $\overline{c} = \Psi_w(\overline{x})$

where M, V, and V_1 are, respectively, the mean value, the variance and the modified variance of Ψ_w.

Algorithm 1. Mean Value of Level Sets for Multiobjective Problems – MVLSM

Data: $\varepsilon \geq 0$, $\overline{w} \in \mathbb{R}^r_+$ with $\overline{w}_i \in (0,1)$ for all $i = 1, \ldots, r$ and $\xi \in \mathbb{R}^r_+ \backslash \{0\}$.

Scalarization

Compute $f^*_\ell = \min_{x \in X}\{f_\ell(x)\}$, for each $\ell \in \{1, \ldots, r\}$ and define $u^* = F^* - \xi$.

Consider the weighting vector $w \in \mathbb{R}^r$ such that $w_i = \dfrac{\overline{w}_i}{\|\overline{w}\|_1}$, for $i = 1, \ldots, r$, and the scalarized function $\Psi_w(x) = \max_\ell\{w_\ell(f_\ell(x) - u^*_\ell)\}$.

Initialization

Take $c_0 \in \mathbb{R}$ such that $H_{c_0} = \{x \in \mathbb{R}^n \mid \Psi_w(x) \leq c_0\} \neq \emptyset$.

Set $k := 0$.

Iterations

REPEAT

Let $H_{c_k} = \{x \in \mathbb{R}^n \mid \Psi_w(x) \leq c_k\}$.

Compute $VF = V_1(\Psi_w, c_k, X) = \dfrac{1}{\mu(H_{c_k} \cap X)} \displaystyle\int_{H_{c_k} \cap X} (\Psi_w(x) - c_k)^2 \, d\mu$.

Compute $c_{k+1} = M(\Psi_w, c_k, X) = \dfrac{1}{\mu(H_{c_k} \cap X)} \displaystyle\int_{H_{c_k} \cap X} \Psi_w(x) \, d\mu$.

$k := k + 1$

UNTIL $VF < \varepsilon$

$\overline{c} = c_k$ and $\overline{H} = H_{\overline{c}}$

Outline

A1′. X is a robust and closed set,

A2′. The functions f_ℓ, $\ell = 1, \cdots, r$, are continuous,

A3′. There exist an index ℓ_0 and $c_0 \in \mathbb{R}$ such that the set $\{x \in X \mid f_{\ell_0}(x) \leq c_0\}$ is compact.

Theorem

Suppose that the problem (MOP) satisfies **A1′**, **A2′**, and **A3′**. Given a weighting vector $w \in W^*$, consider the sequence $\{c_k\}$ generated by Algorithm MVLSM. Then, this sequence is convergent and the limit $\overline{c} = \lim_{k\to\infty} c_k$ is the global minimum value of Ψ_w over X. Furthermore, $H_{\overline{c}} \cap X$ is the set of its global minimizers and consequently a subset of weak Pareto optimal solutions of (MOP).

Some considerations

- $H_{c_0} \cap X$ is nonempty
- $X = D$ is a cuboid in $\mathbb{R}^n$ given by $D = \{x \in \mathbb{R}^n \mid a \leq x \leq b\}$.
- $\rho_k = \rho(H_{c_k}) = \max_{x,y \in H_{c_k}} ||x - y|| \to 0$ when $k \to \infty$.
- $\overline{c} = \min_{x \in D} f(x) = \min_{x \in H_{c_k} \cap D} f(x) = \min_{x \in D_k} f(x)$

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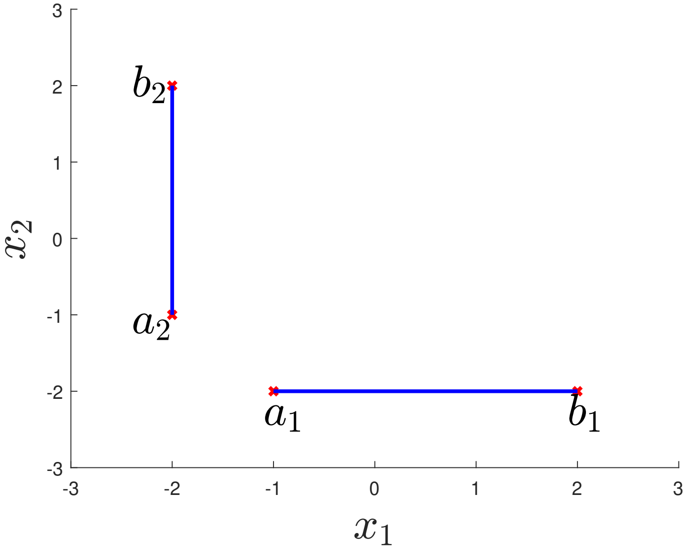

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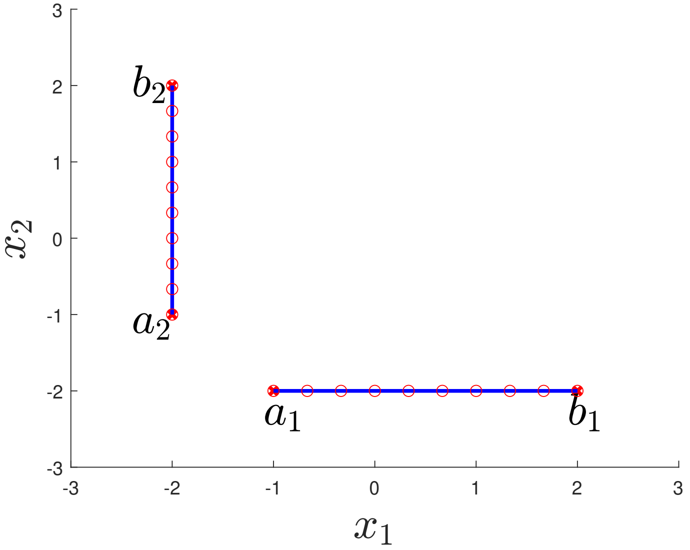

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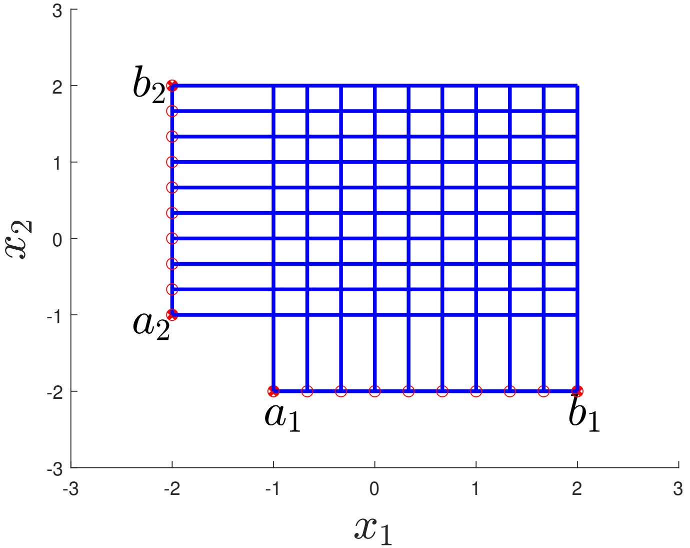

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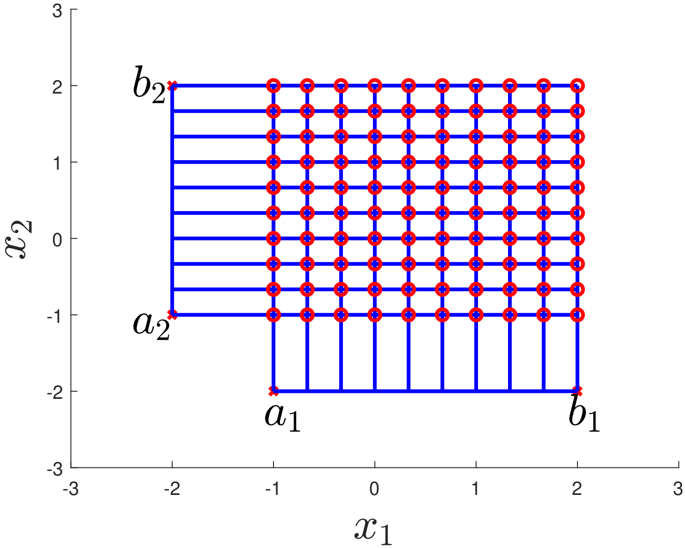

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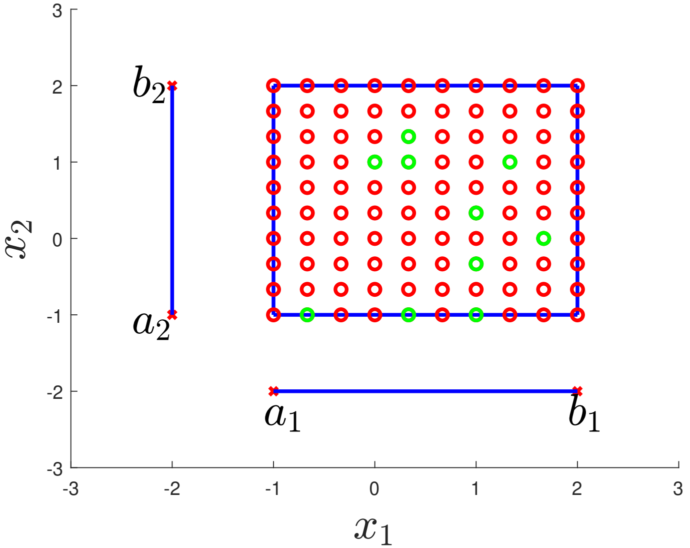

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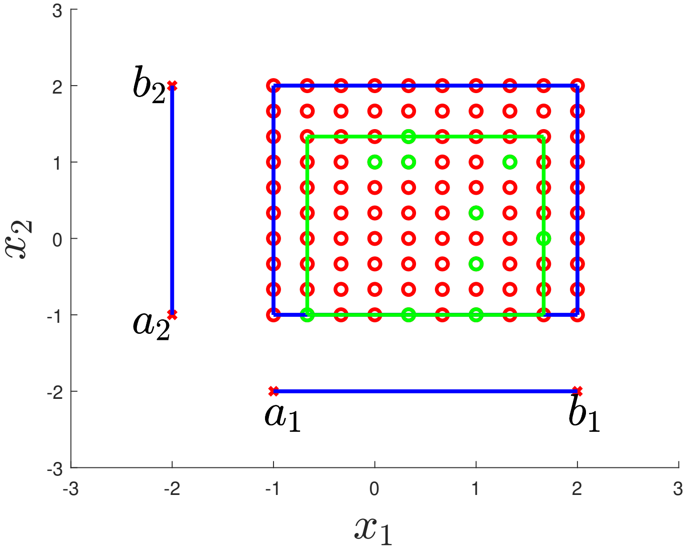

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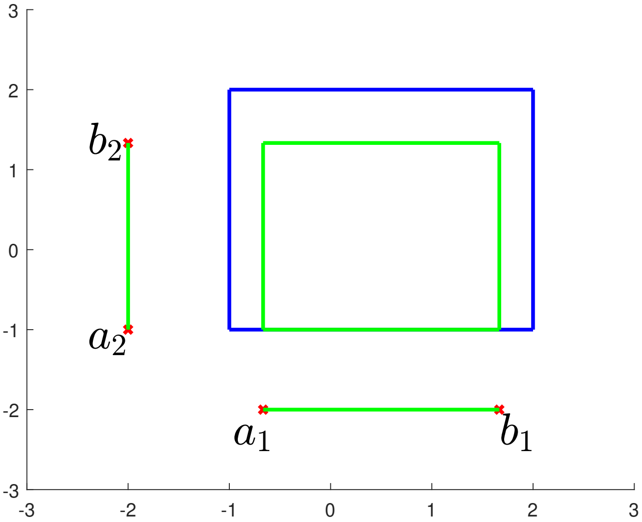

Outline

Numerical Settings

- High performance workstation MARKOV: 2*CPU Intel® Xeon® Processor E5-2650 v3 (10 Cores, 25M Cache, 2.30 GHz), 160GB RAM 2,133GHz
- Matlab 2018b
- $c_0 = 10^8$
- $\varepsilon = 10^{-8}$
- $\xi_\ell = 10^{-4}$, $\ell = 1, \dots, r$
- 26 unconstrained and box-constrained multiobjective problems - dimension at most 4 - presented in Chankong (1983), Deb (2001), Deb (2005), and Veldhuizen (1999)
- Primary results - 3000 runs

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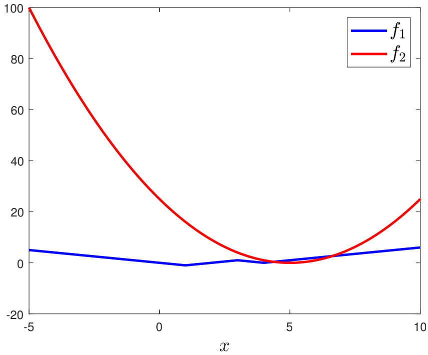
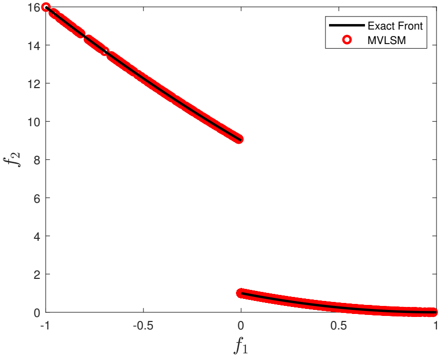
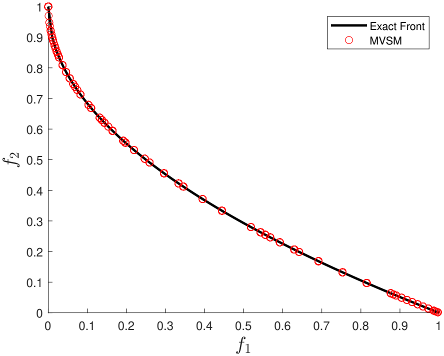
Problem MOP 14 - Veldhuizen, 1999

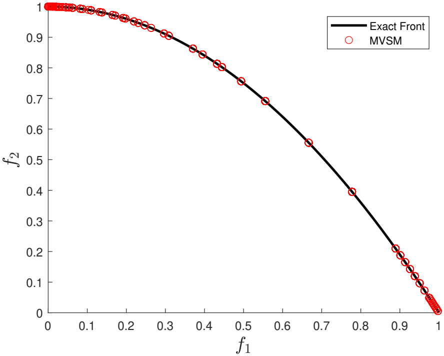




$\overline{T} = 0.0867$ seconds and $\overline{k} = 45.7$

Problem ZDT 1 and ZDT 2 - Deb, 2001





$\overline{T} = 0,012$ seconds and $\overline{k} = 3,5$

$\overline{T} = 0,011$ seconds and $\overline{k} = 3,3$

Numerical Settings

- High performance workstation
- Matlab 2018b
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- $\varepsilon = 10^{-8}$
- $\xi_\ell = 10^{-4}$
- 26 problems
- Secondary results - maximum of 20000 function evaluations
- Comparison among MVLSM, MOIF, and DMS
  - DMS → A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente. *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109-1140
  - MOIF → G. Cocchi, G. Liuzzi, A. Papini, and M. Sciandrone. *An implicit filtering algorithm for derivative-free multiobjective optimization with box constraints*, Comput. Optim. Appl. (2018), 69, 267–296
- **MOIF** and **DMS**: default values

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

  * Percentage of points generated in the reference Pareto front

- Hypervolume

$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

  * Volume of the dominated region

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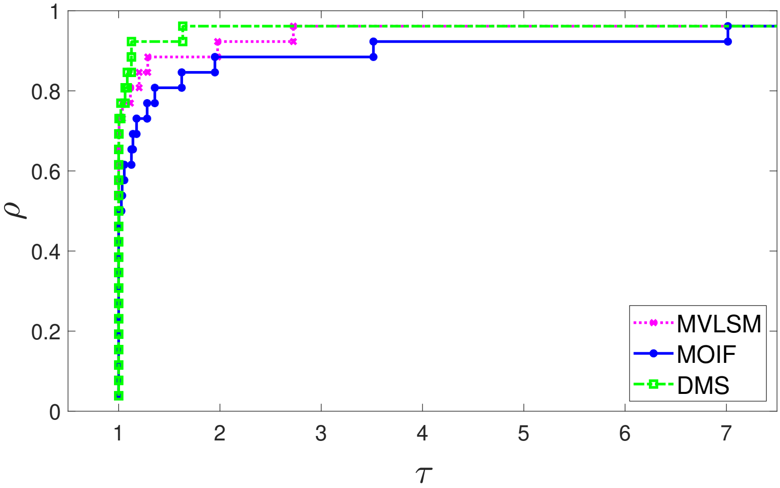
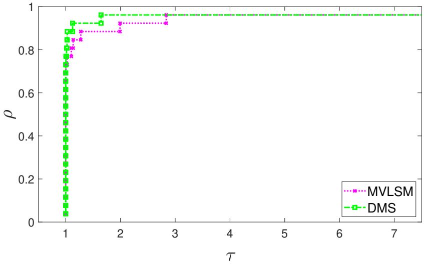
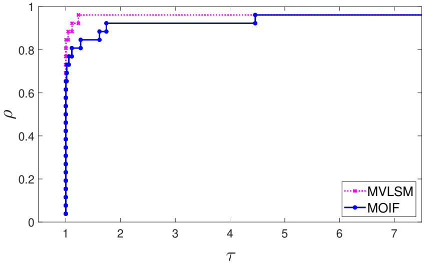
Results - Purity

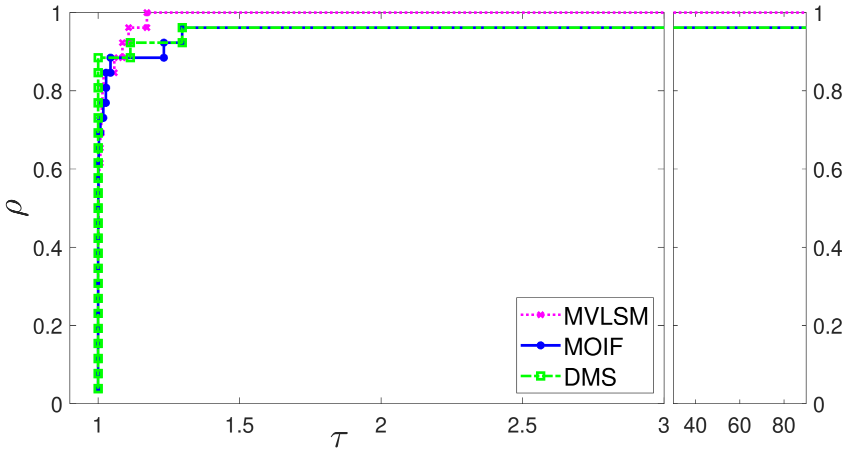




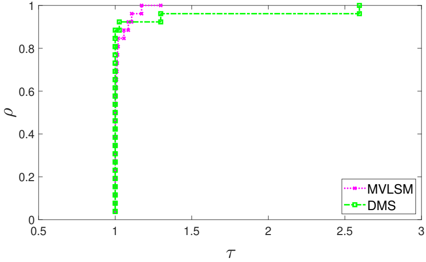


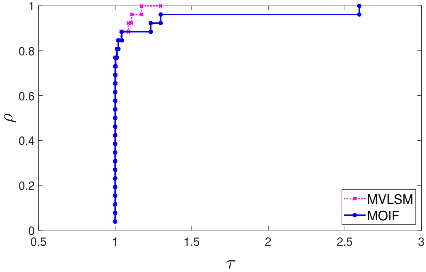
- MVLSM
- MOIF
- DMS

Results - Hypervolume



Results - Hypervolume





- MVLSM
- MOIF
- DMS

Outline

Conclusions

- Global optimality integral conditions were extended to multiobjective optimization
- An algorithm was proposed for multiobjective optimization, based on a Chebyshev weighted scalarization and its convergence was established
- Preliminary numerical tests were performed to illustrate its effectiveness
- The conditions are stated in terms of multiple integrals, which may narrow the application such a theory to problems with many variables
- The points in the Pareto front approximation are not well spread

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Future Work

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Everton J. Silva[a], Elizabeth W. Karas[b], and Lucelina B. Santos[b]

- Implementing efficient methods to compute integrals with many variables
- Smarter choices of the weights trying to generate points well spread in the Pareto front approximation
- Explore other scalarization techniques

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THANKS FOR YOUR ATTENTION!

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