A Direct Multisearch Inexact Restoration Filter Method for Biobjective Optimization

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Outline

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Multiobjective Constrained Derivative-free Optimization

$$\min_{x \in \Omega \subset \mathbb{R}^n} \quad F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^\top$$

$$f_j : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}, \;\; j = 1, 2, \ldots, m \geq 2$$

with $\Omega = X \cap \{x \in \mathbb{R}^n \mid C(x) \leq 0\}$, where X is a full dimensional polyhedron and $C : \mathbb{R}^n \to (\mathbb{R} \cup \{+\infty\})^p$

- several conflicting objectives
- impossible to use or approximate derivatives of the objective function
- expensive objective function evaluation

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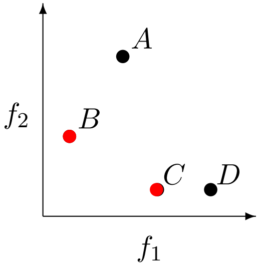
Direct MultiSearch (DMS) Main Lines

- does not aggregate any of the objective function components
- makes use of Pareto dominance

Pareto Dominance (x dominates y)

$F(x) \leq F(y)$, with $F(x) \neq F(y)$



Pareto Front: $\{B, C\}$

- generalizes directional direct-search to MOO
- considers the search/poll paradigm with an optional search step
- computes approximations to the complete Pareto front

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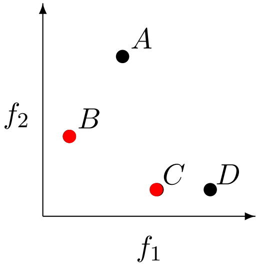

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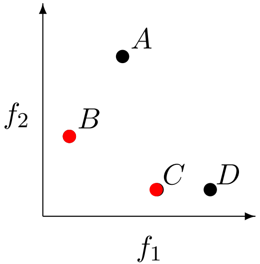

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Direct MultiSearch (DMS) Main Lines

- constraints are addressed by an extreme barrier approach

$$F_\Omega(x) = \begin{cases} F(x) & \text{if } x \in \Omega, \\ (+\infty, +\infty, \ldots, +\infty)^\top & \text{otherwise} \end{cases}$$

- keeps a list of feasible nondominated points
- poll centers are chosen from the list
- successful iterations correspond to list changes

successful iteration $\Leftrightarrow$ new feasible nondominated point

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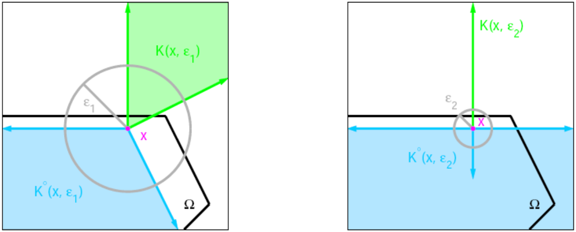
Poll Directions and Constraints

- Unconstrained or variable bounds: coordinate search

$$D = D_{\oplus} = [\, I \;\; -I \,]$$

- Linear constraints: directions must conform to the geometry of nearby constraints



Abramson, Brezhneva, Dennis, and Pingel [2008]

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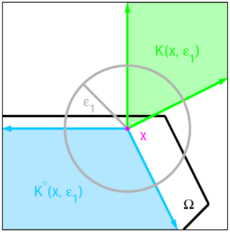

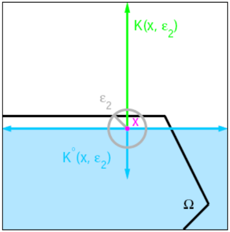

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Motivation

- DMS $\rightarrow$ A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente, *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109–1140

- DFMO $\rightarrow$ G. Liuzzi, S. Lucidi, and F. Rinaldi, *A derivative-free approach to constrained multiobjective nonsmooth optimization*, SIAM J. Optim. (2016), 26, 2744–2774

- 93 biobjective problems with nonlinear constraints and variable bounds
  - number of variables between 3 and 30
  - number of nonlinear constraints between 1 and 29
  - maximum of 20000 function evaluations

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

- Spreads Γ and Δ

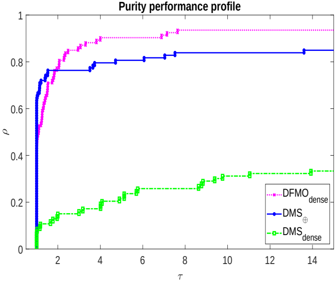
$$\Gamma_{p,s} = \max_{j \in \{1,\ldots,m\}} \left(\max_{i \in \{0,\ldots,N\}} \{d_i\} \right)$$

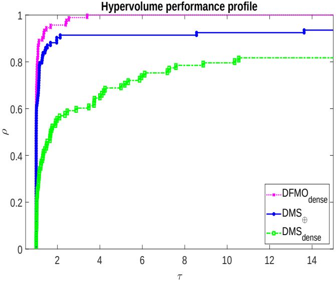
$$\Delta = \max_{j \in \{1,\ldots,m\}} \left(\frac{d_0 + d_N + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_0 + d_N + (N-1)\bar{d}} \right)$$

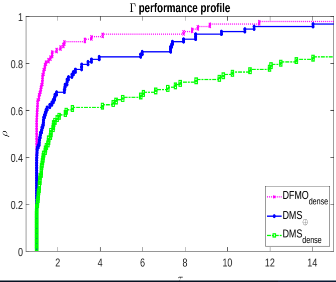
- Hypervolume

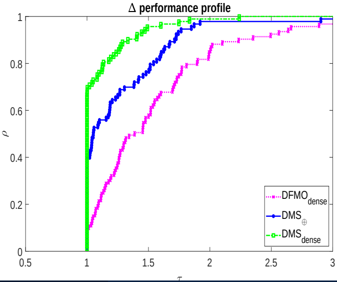
$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

DFMO – Liuzzi, Lucidi, and Rinaldi (2016)









Outline

Problem Reformulation – Filter Approach

$$\min_{x \in X} \, (F(x); h(x)) = (f_1(x), f_2(x), \ldots, f_m(x), h(x))^\top$$

where X is a full dimensional polyhedron and

$$h(x) = \|C(x)_+\|_2^2 = \sum_{i=1}^{p} \max\{0, c_i(x)\}^2$$

Constraints in X continue to be addressed by an extreme barrier approach.

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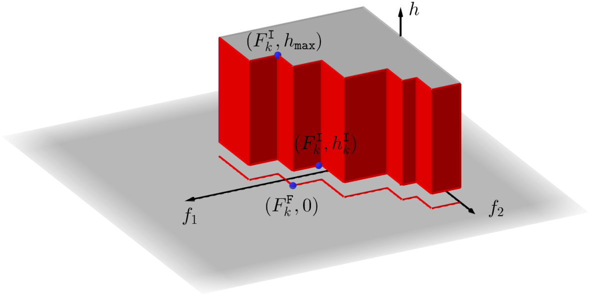
New Dominance Relation

A point x' is filtered by $\mathcal{F}$ if:

- $h(x') > h_{\mathtt{max}}$ (for some fixed $h_{\mathtt{max}} > 0$)
- there is $x \in \mathcal{F}$ such that $x' \succeq_{(F;h)} x$
- $x' \notin \Omega$ and there is $x \in \mathcal{F}$ such that

$$h(x') \geq h(x) \text{ and } x' \succeq_F x$$



DMS Filter and Inexact Restoration Approach

- Nonlinear feasibility is treated as an additional objective
- Priority given to feasible poll centers
- When all poll points associated with a poll center x_k are infeasible, switches to an infeasible poll center

Attempts to restore feasibility by solving:

$$\min_{y \in X} \quad \frac{1}{2}\|y - x_k\|^2$$
$$\text{s.t.} \quad h(y) \leq \xi(\alpha_k) h(x_k),$$

where $\xi : (0, +\infty) \to (0, 1)$, is continuous, nondecreasing, and satisfies

$$\xi(t) \to 0 \text{ when } t \downarrow 0$$

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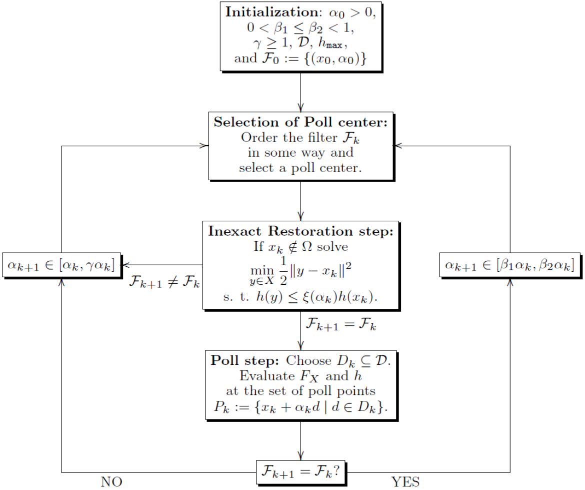
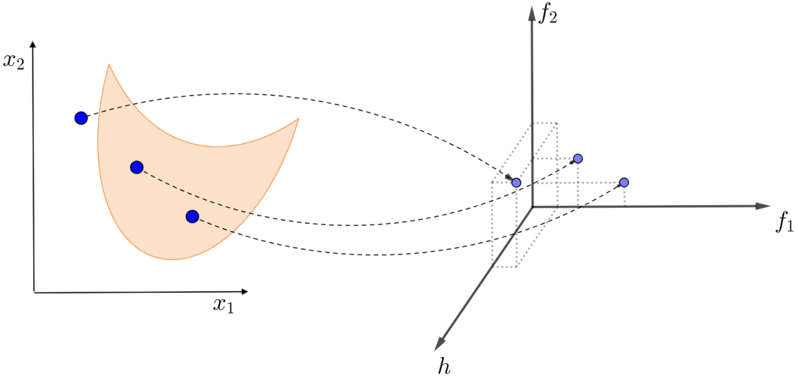
DMS-FILTER-IR – Algorithmic Structure



Solutions: $L := \{(x, \alpha) \in \mathcal{F} \mid (F(x); h(x)) = (F(x); 0)\}$

DMS-FILTER-IR – Poll Step



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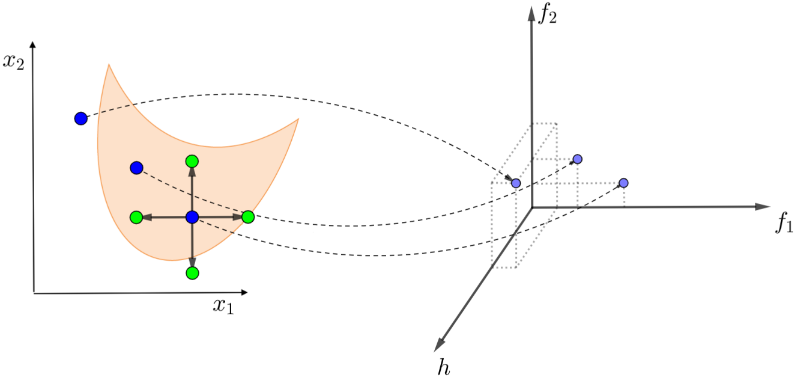

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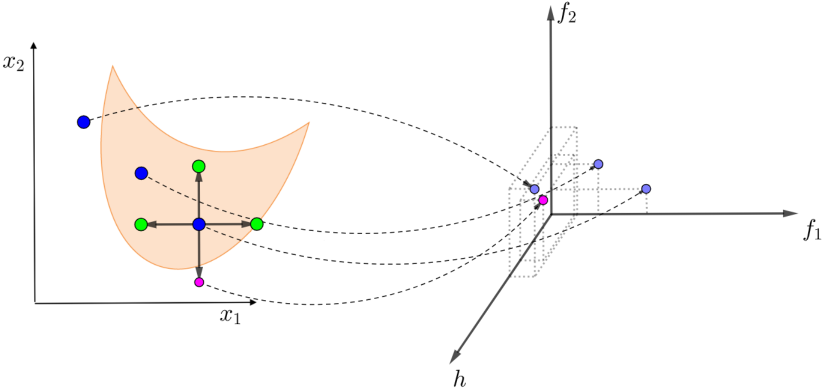

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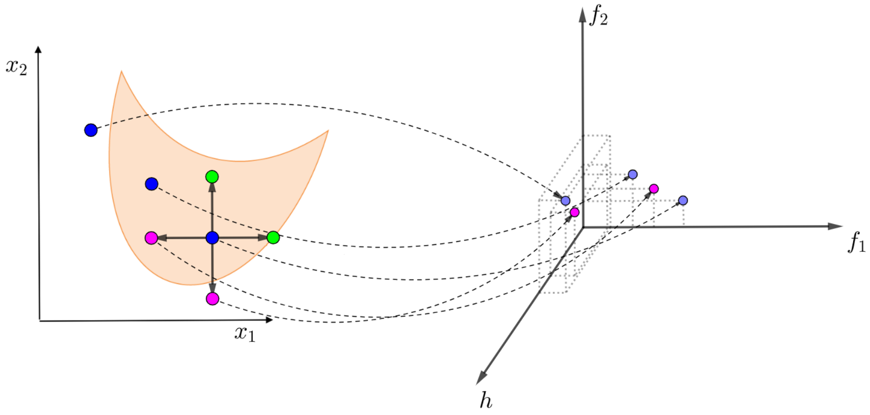

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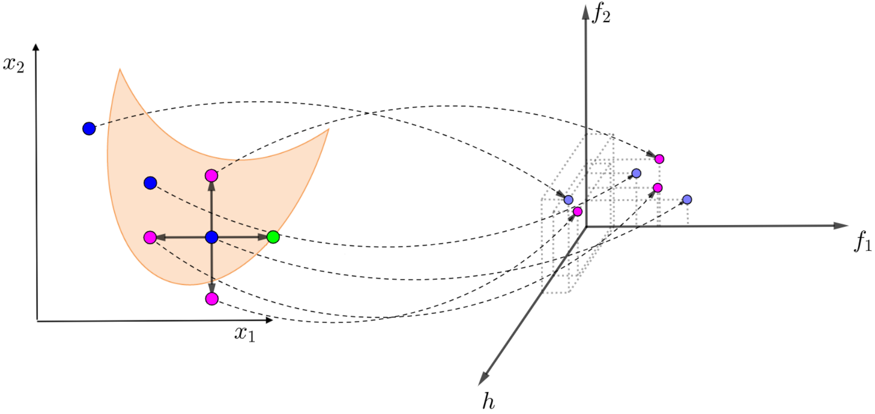

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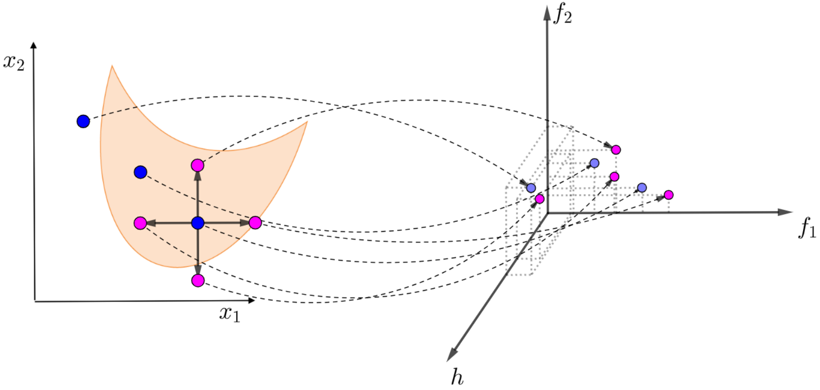

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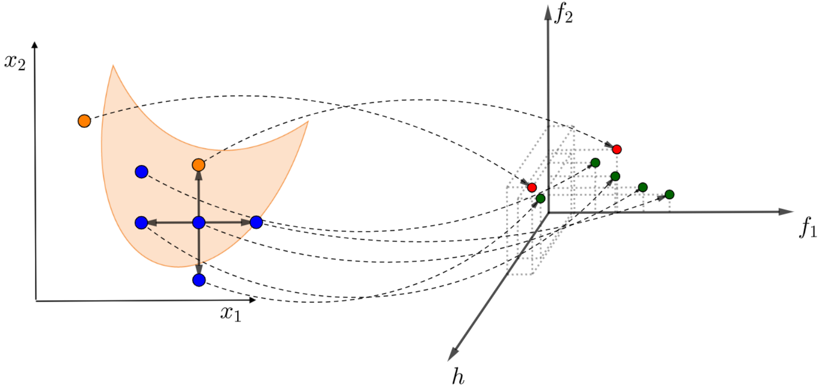

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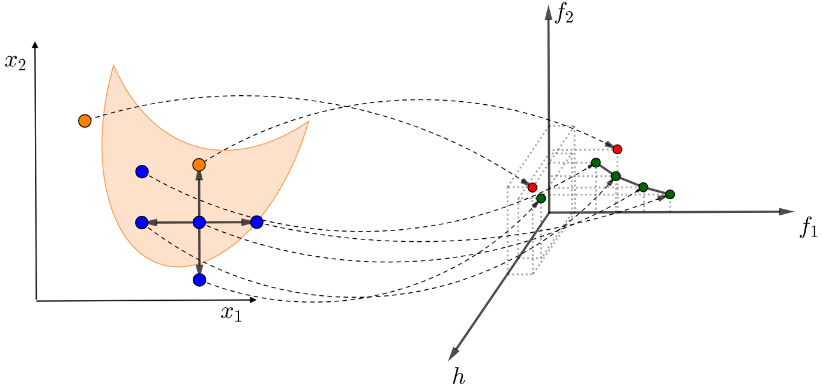
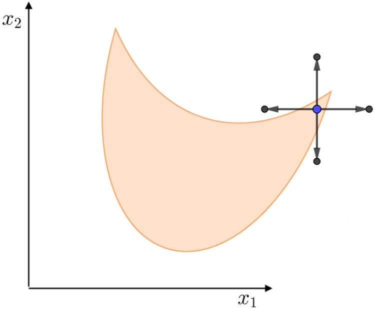


Poll Center Selection

Feasible to Infeasible

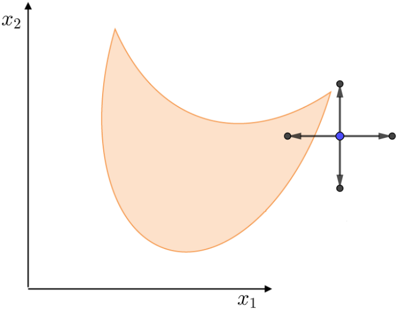
Polling only generates infeasible points



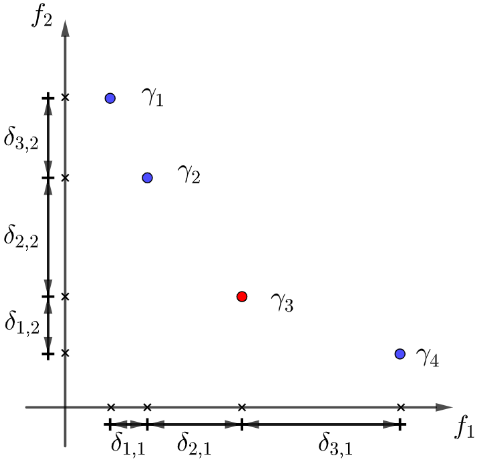
Infeasible to Feasible

Infeasible x_k generates feasible point by inexact restoration or polling

$$\begin{aligned} \min_{y \in X} \quad & \frac{1}{2}\|y - x_k\|^2 \\ \text{s.t.} \quad & h(y) \leq \xi(\alpha_k) h(x_k) \end{aligned}$$



Feasible Poll Center – Most Isolated Point



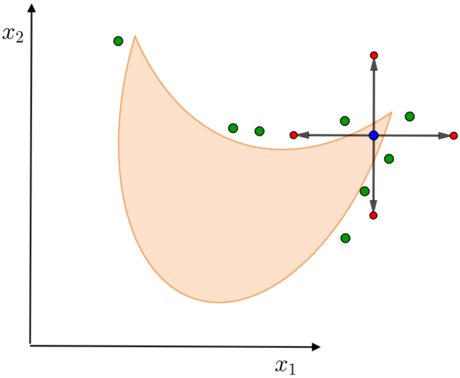
$$\gamma_1 = \frac{\delta_{3,2} + \delta_{1,1}}{2}$$

$$\gamma_2 = \frac{\frac{\delta_{3,2}+\delta_{2,2}}{2} + \frac{\delta_{1,1}+\delta_{2,1}}{2}}{2}$$

$$\gamma_3 = \frac{\frac{\delta_{2,2}+\delta_{1,2}}{2} + \frac{\delta_{2,1}+\delta_{3,1}}{2}}{2}$$

$$\gamma_4 = \frac{\delta_{1,2} + \delta_{3,1}}{2}$$

Infeasible Poll Center



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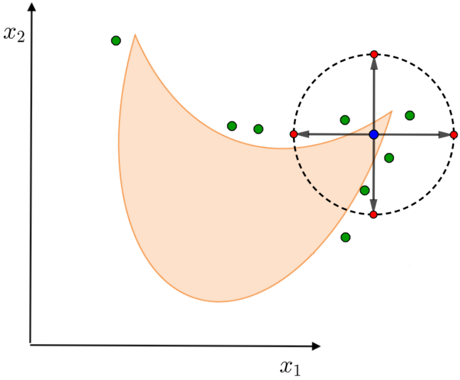

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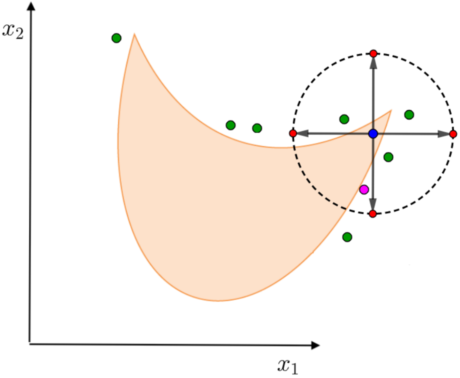

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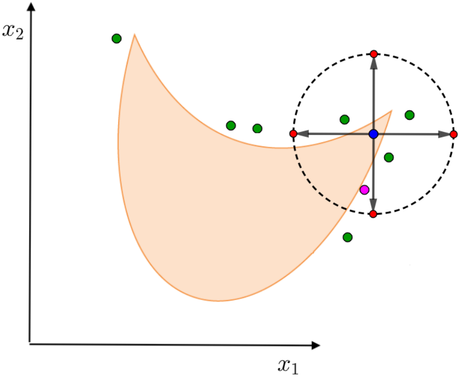

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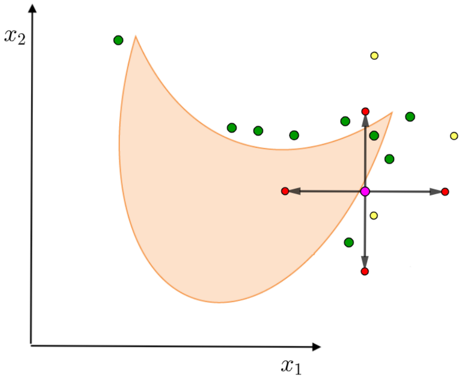


Outline

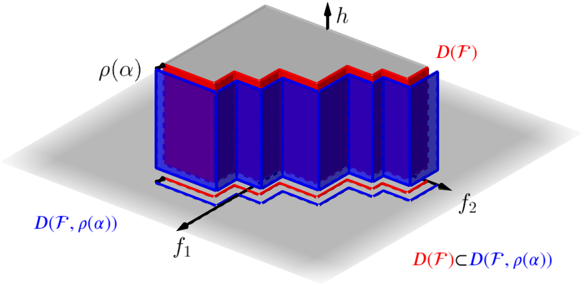
Globalization Strategies

Using Integer Lattices (Torczon [1997], Audet and Dennis [2002])

- requires only simple decrease
- poll directions and step size must satisfy integer/rational requirements

Imposing Sufficient Decrease (Kolda, Lewis, and Torczon [2003])

- use of a forcing function
  $\rho : (0, +\infty) \to (0, +\infty)$, continuous, nondecreasing, and satisfying
  $\rho(t)/t \to 0$ when $t \downarrow 0$
- x is nondominated $\Leftrightarrow (F_X(x), h(x)) \notin D(\mathcal{F}, \rho(\alpha))$



Convergence Results – Sequences

Linked sequence

A sequence $\{(x_k, \alpha_k)\}_{k \in K}$ such that (x_k, α_k) is generated from (x_{k-1}, α_{k-1}) at a poll or at an inexact restoration step.

Refining sequence

A sequence $\{(x_k, \alpha_k)\}_{k \in K}$, such that $k \in K$ is an unsuccessful iteration and $\lim_{k \in K} \alpha_k = 0$.

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Theorem

From any linked sequence it is possible to extract a convergent refining subsequence (under any of the two globalization strategies).

Let $\overline{x}$ be the limit point of a convergent refining subsequence $\{x_k\}_{k \in K}$.

Definition (Refining Directions)

Refining directions for $\overline{x}$ are limit points of $\{d_k/\|d_k\|\}_{k \in K}$, where $d_k \in D_k$ and $x_k + \alpha_k d_k \in \mathcal{S} := \{x \in X \mid h(x) \leq h_{\max}\}$.

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Consider a linked sequence $\{(x_k, \alpha_k)\}_{k\in K}$ generated by DMS-FILTER-IR. Let $\{(x_k^{\mathtt{F}}, \alpha_k^{\mathtt{F}})\}_{k\in K'\subseteq K}$ be a feasible refining subsequence converging to $\overline{x} \in \Omega$. Assume that F and h are Lipschitz continuous near $\overline{x}$.

Theorem

- If $d \in \mathrm{int}(T_\Omega^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m\} \text{ such that } \quad f_j^\circ(\overline{x}; d) \geq 0$$

- If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_\Omega^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point of F (in Ω):

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Convergence Results – Infeasible case

Consider a linked sequence $\{(x_k, \alpha_k)\}_{k \in K}$ generated by DMS-FILTER-IR. Let $\{(x_k^{\mathtt{I}}, \alpha_k^{\mathtt{I}})\}_{k \in K' \subseteq K}$ be a refining subsequence converging to $\overline{x} \in \mathcal{S}$. Assume that F and h are Lipschitz continuous near $\overline{x}$.

Theorem

- If $d \in \mathrm{int}(T_{\mathcal{S}}^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

- If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_{\mathcal{S}}^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point of F in $\mathcal{S} := \{x \in X \mid h(x) \leq h_{\mathtt{max}}\}$:

$$\forall d \in T_{\mathcal{S}}^{Cl}(\overline{x}), \exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

Outline

Numerical Settings

- Comparison among **DFMO**, **DMS-EB**, **DmultiMads-PB*** and **DMS-FILTER-IR**
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of nonlinear constraints between 1 and 29
- Initialization:
  - **DMS-EB:** Feasible point provided by Karmitsa [2007]
  - **Others:** n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS-EB** and **DMS-FILTER-IR**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO** and **DmultiMads-PB**:
    - default values
  - **All:** maximum of 500, 5000, and 20000 function evaluations

* **DmultiMads-PB** → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *Handling of constraints in multiobjective blackbox optimization*. ArXiv:2204.00904

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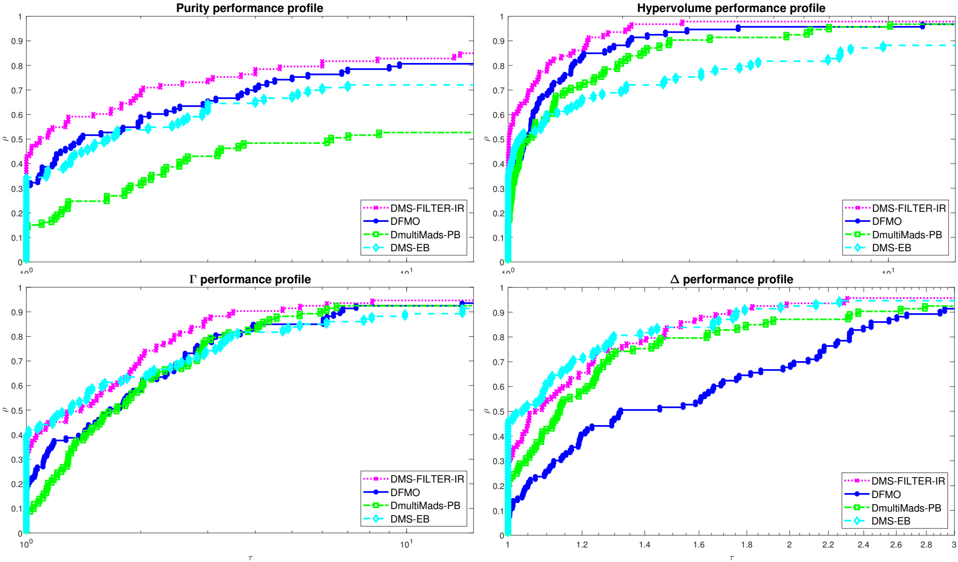
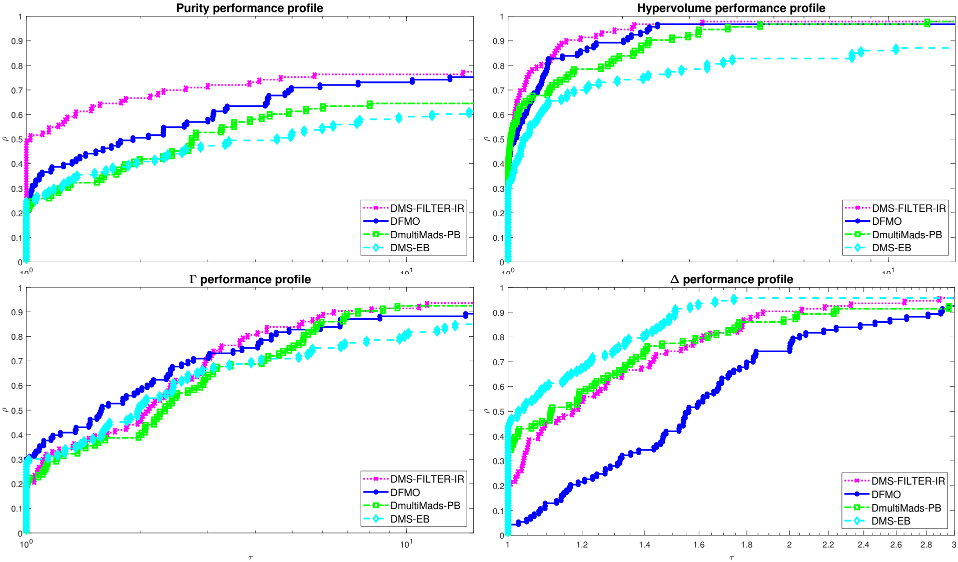
Results - 500 function evaluations



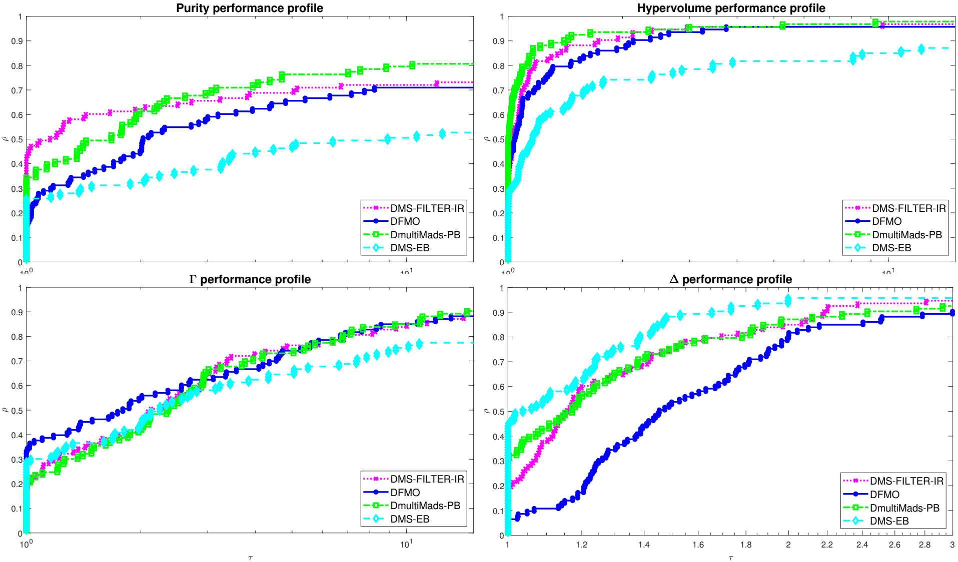
DMS-FILTER-IR DFMO DmultiMads-PB DMS-EB

Results - 5000 function evaluations



DMS-FILTER-IR **DFMO** **DmultiMads-PB** **DMS-EB**

Results - 20000 function evaluations



DMS-FILTER-IR DFMO DmultiMads-PB DMS-EB

Outline

Conclusions and Future Work

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- DMS-FILTER-IR extends filter methods, with an inexact restoration step, to the DMS framework
- DMS-FILTER-IR presents a well-supported convergence analysis
- DMS-FILTER-IR presents competitive numerical results for constrained biobjective derivative-free optimization problems

Future Work

- developing a competitive numerical implementation for problems with more than two objectives

Thank you for your attention!

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