An Inexact Restoration Direct Multisearch Filter Approach to Constrained Optimization

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Outline

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1. Introduction
2. DMS-FILTER-IR
3. Convergence Analysis
4. Numerical Results
5. Conclusions and Future Work

Multiobjective Constrained Derivative-free Optimization

$$\min_{x \in \Upsilon \subset \mathbb{R}^n} \quad F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^\top$$

$$f_j : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}, \; j = 1, 2, \ldots, m \geq 2$$

with $\Upsilon = \Omega \cap X$ (where: Ω relaxable and X unrelaxable)

- several conflicting objectives
- impossible to use or approximate derivatives of the objective function
- expensive objective function evaluation

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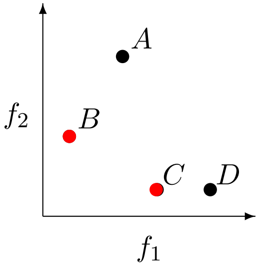
Direct MultiSearch (DMS) Main Lines

- does not aggregate any of the objective function components
- makes use of Pareto dominance



Pareto Front: $\{B, C\}$

Pareto Dominance (x dominates y)

$$F(x) \le F(y), \text{ with } F(x) \ne F(y)$$

- generalizes directional direct-search to MOO
- considers the search/poll paradigm with an optional search step
- computes approximations to the complete Pareto front

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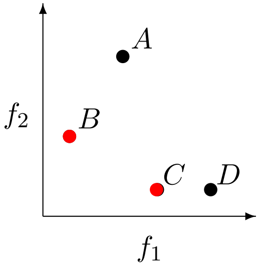

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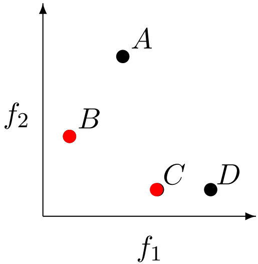

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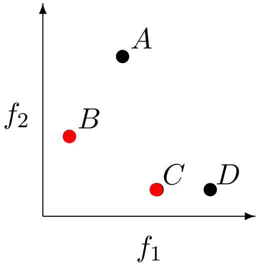

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- constraints are addressed by an extreme barrier approach

$$F_{\Upsilon}(x) = \begin{cases} F(x) & \text{if } x \in \Upsilon, \\ (+\infty, +\infty, \ldots, +\infty)^{\top} & \text{otherwise} \end{cases}$$

- keeps a list of feasible nondominated points
- poll centers are chosen from the list
- successful iterations correspond to list changes

successful iteration $\Leftrightarrow$ new feasible nondominated point

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Motivation

- DMS $\rightarrow$ A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente, *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109–1140
- DFMO $\rightarrow$ G. Liuzzi, S. Lucidi, and F. Rinaldi, *A derivative-free approach to constrained multiobjective nonsmooth optimization*, SIAM J. Optim. (2016), 26, 2744–2774
- 93 biobjective problems with nonlinear constraints and variable bounds
  - number of variables between 3 and 30
  - number of nonlinear constraints between 1 and 29
  - maximum of 20000 function evaluations

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

- Spreads Γ and Δ

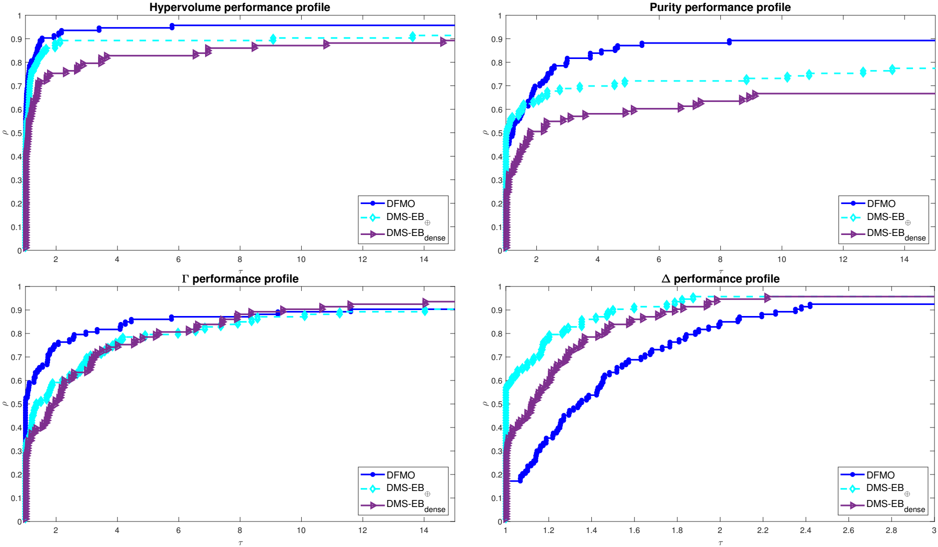
$$\Gamma_{p,s} = \max_{j \in \{1,\ldots,m\}} \left(\max_{i \in \{0,\ldots,N\}} \{d_i\} \right)$$

$$\Delta = \max_{j \in \{1,\ldots,m\}} \left(\frac{d_0 + d_N + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_0 + d_N + (N-1)\bar{d}} \right)$$

- Hypervolume

$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

Nonlinear + Bound Constraints (Biobjective Problems)



DFMO **DMS-EB-Coordinate** **DMS-EB-Dense**

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Problem Reformulation – Filter Approach

$$\min_{x \in X} \; (F(x), h(x)) = (f_1(x), f_2(x), \ldots, f_m(x), h(x))^\top$$

where X is the set of unrelaxable constraints and

$$h(x) = \|C(x)_+\|_2^2 = \sum_{i=1}^{p} \max\{0, c_i(x)\}^2$$

Constraints in X continue to be addressed by an extreme barrier approach, and it is assumed $x_0 \in X$.

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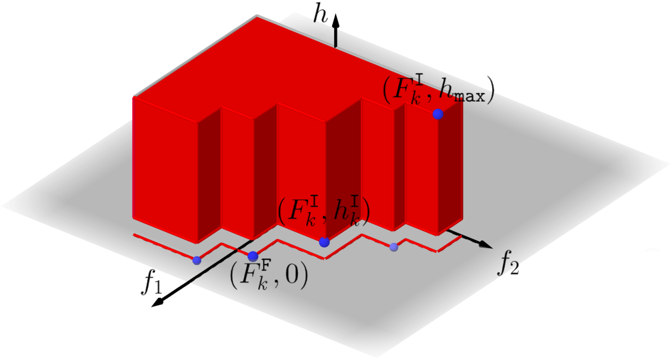
List of Nondominated Points

A point x' is said to be filtered by the list L if any of the following properties hold:

- $h(x') > h_{\mathtt{max}}$ (for some fixed $h_{\mathtt{max}} > 0$)
- there is $x \in L$ such that $(F(x), h(x)) \leq (F(x'), h(x'))$ with $(F(x), h(x)) \neq (F(x'), h(x'))$



DMS Filter and Inexact Restoration Approach

- Relaxable feasibility is treated as an additional objective
- Priority given to feasible poll centers
- When all poll points associated with a poll center x_k are infeasible, switches to an infeasible poll center

  Attempts to restore feasibility by solving:

$$\begin{aligned} \min_{y \in X} \quad & \tfrac{1}{2}\|y - x_k\|^2 \\ \text{s.t.} \quad & h(y) \le \xi(\alpha_k) h(x_k), \end{aligned}$$

where $\xi : (0, +\infty) \to (0, 1)$, is continuous, and satisfies

$$\xi(t) \to 0 \text{ when } t \downarrow 0$$

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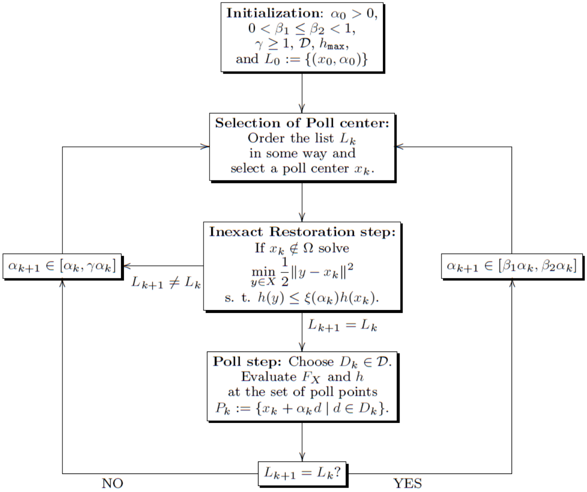
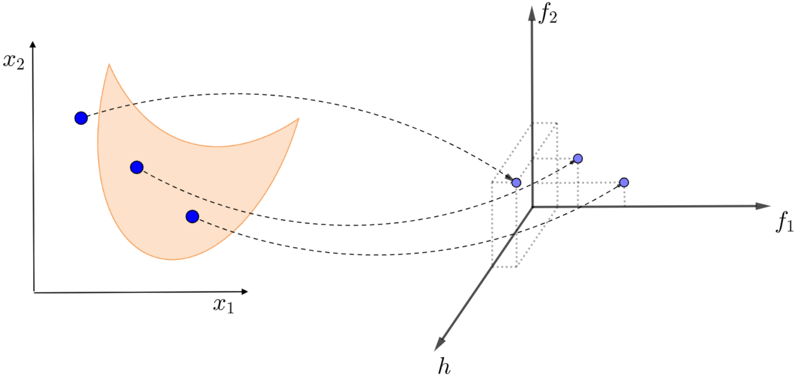
DMS-FILTER-IR – Algorithmic Structure



Solutions$:= \{(x, \alpha) \in L \mid (F(x), h(x)) = (F(x), 0)\}$

DMS-FILTER-IR – Poll Step



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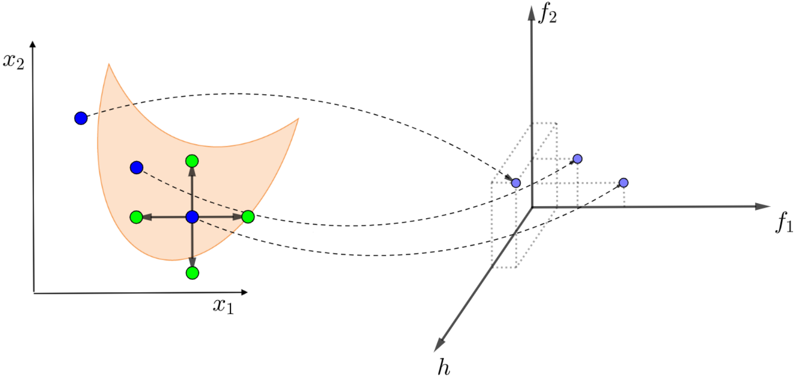

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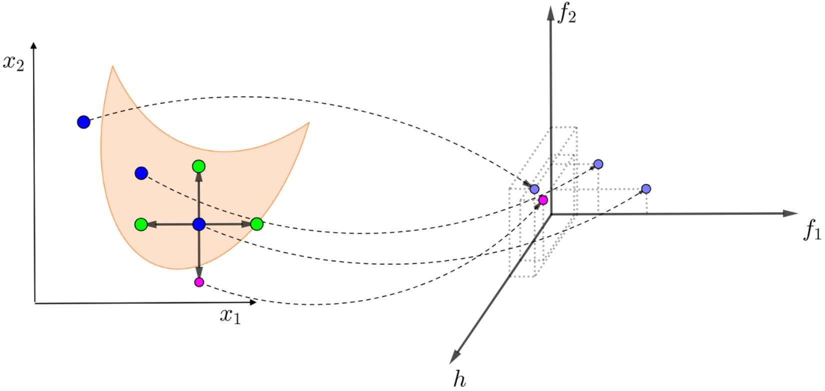

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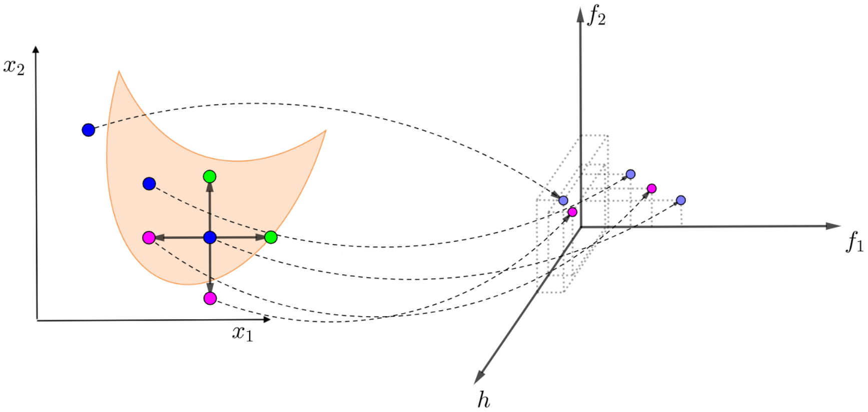

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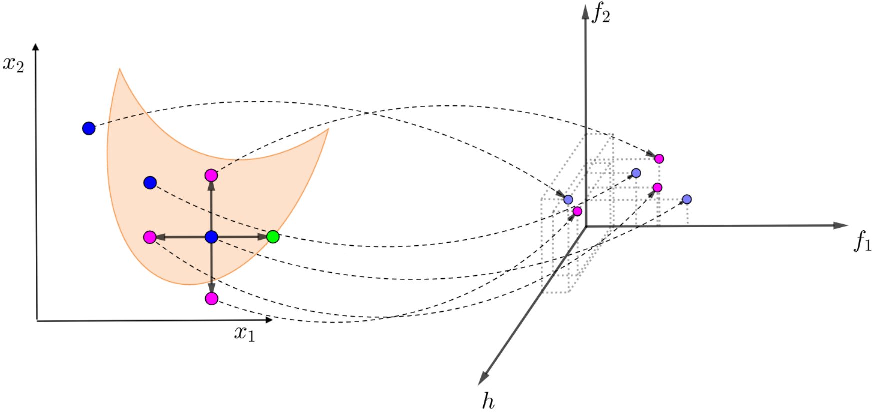

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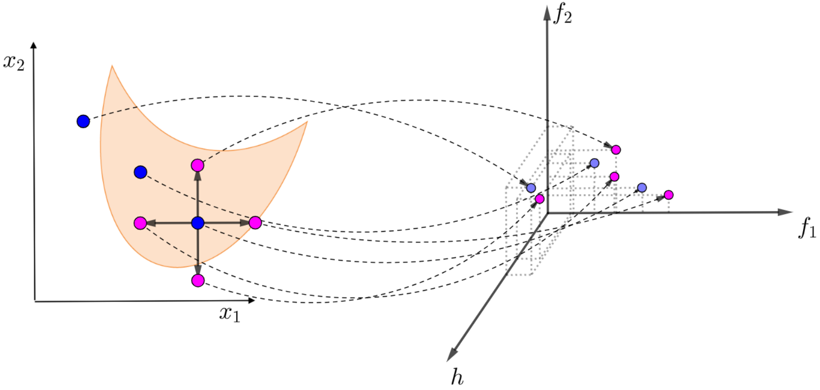

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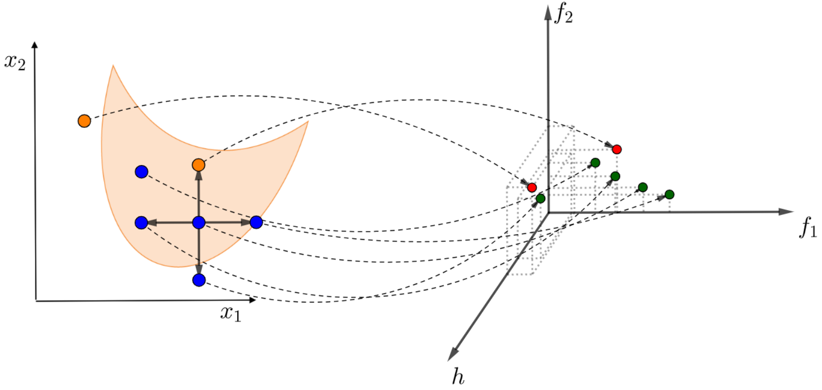

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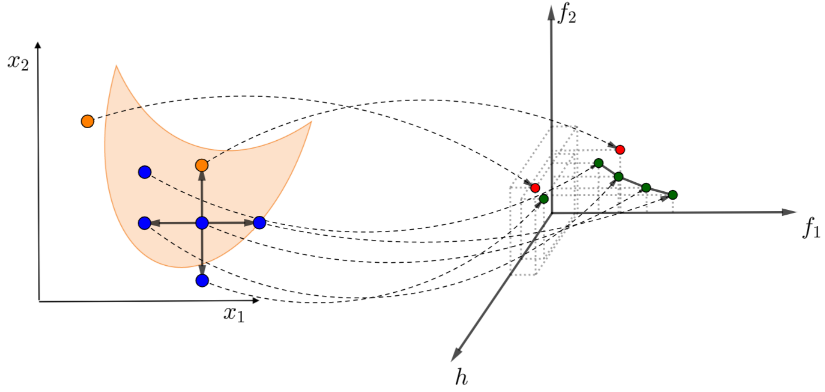
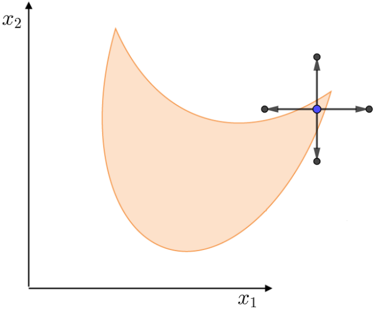


Poll Center Selection

Feasible to Infeasible

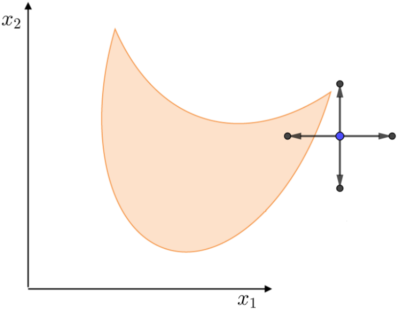
Polling only generates infeasible points



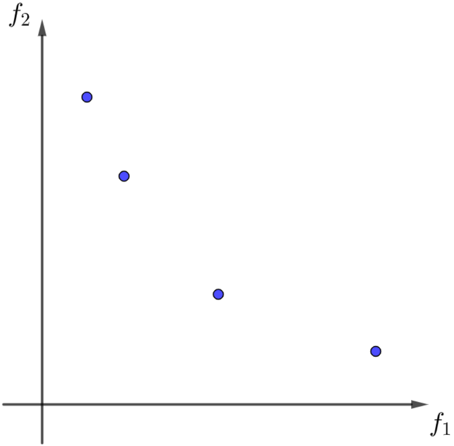
Infeasible to Feasible

Infeasible x_k generates feasible point by inexact restoration or polling

$$\begin{aligned} \min_{y \in X} \quad & \frac{1}{2}\|y - x_k\|^2 \\ \text{s.t.} \quad & h(y) \leq \xi(\alpha_k) h(x_k) \end{aligned}$$



Feasible poll center - Most Isolated Point



- $L \cap \Upsilon$

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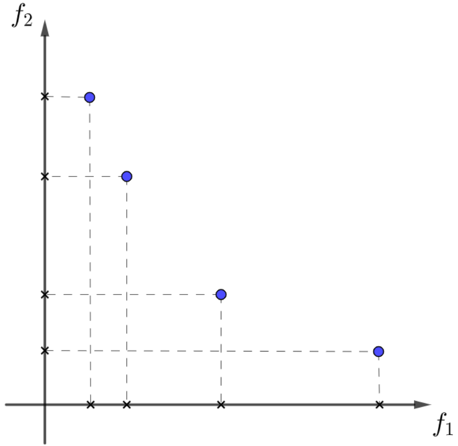

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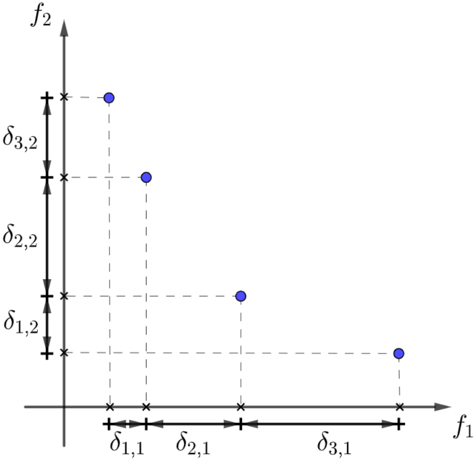
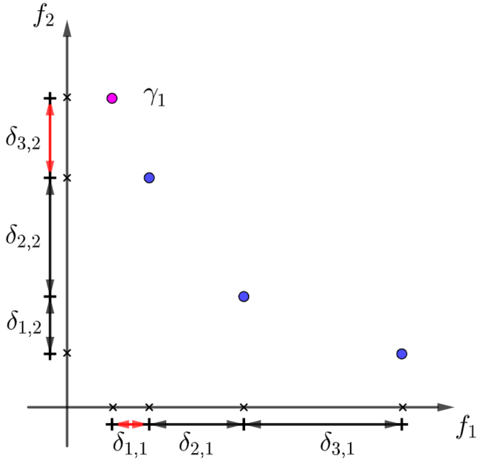


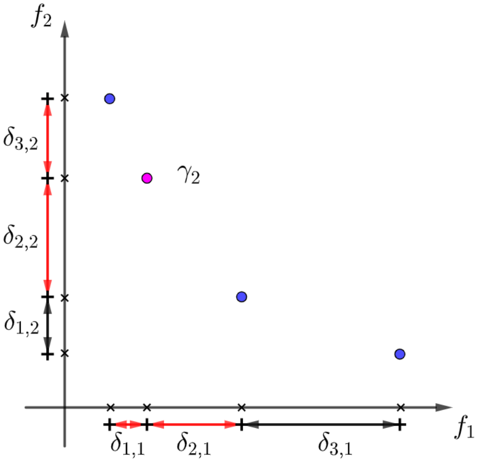
$\delta_{i,j} = f_{i+1,j} - f_{i,j}$
for $i = 1, 2, 3$ and $j = 1, 2$.

Feasible poll center - Most Isolated Point



$$\gamma_1 = \frac{\delta_{3,2} + \delta_{1,1}}{2}$$

Feasible poll center - Most Isolated Point



$$\gamma_2 = \frac{\frac{\delta_{3,2}+\delta_{2,2}}{2} + \frac{\delta_{1,1}+\delta_{2,1}}{2}}{2}$$

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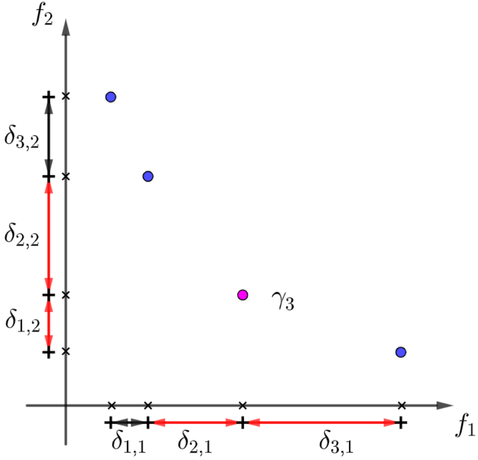

$$\gamma_3 = \frac{\frac{\delta_{2,2}+\delta_{1,2}}{2} + \frac{\delta_{2,1}+\delta_{3,1}}{2}}{2}$$

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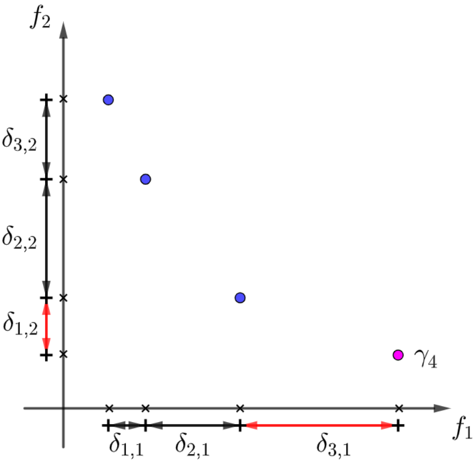
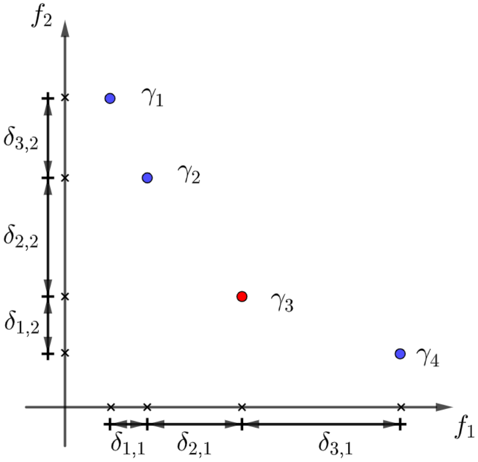


$$\gamma_4 = \frac{\delta_{1,2} + \delta_{3,1}}{2}$$

Feasible poll center - Most Isolated Point



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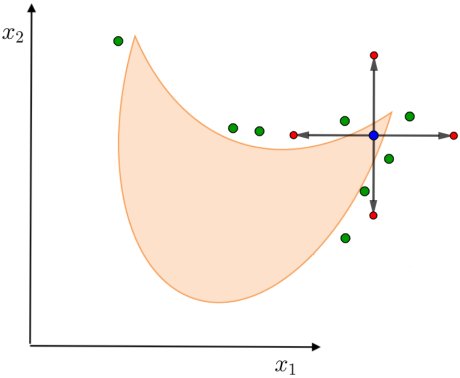
Infeasible Poll Center



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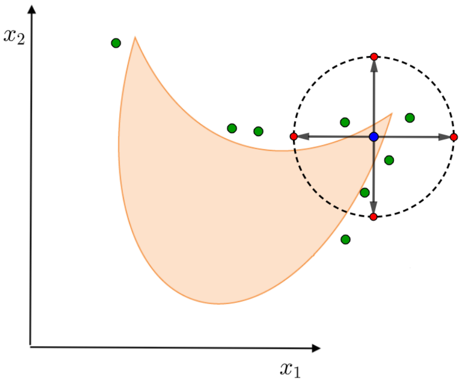

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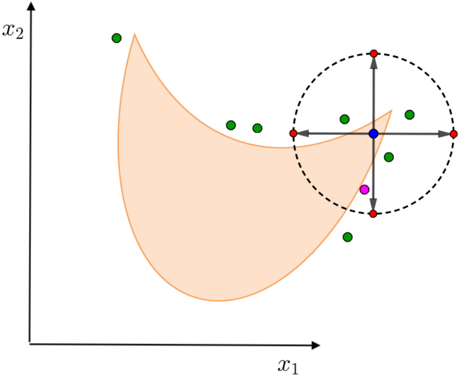

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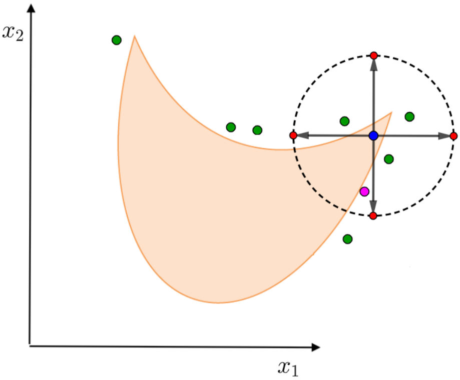

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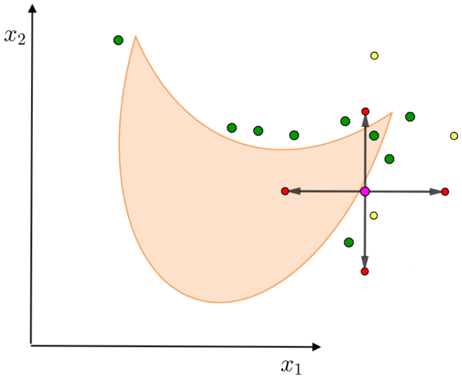


Outline

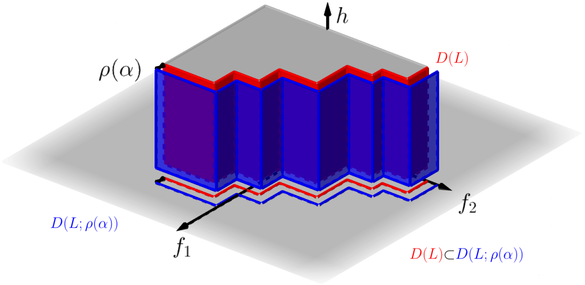
Globalization Strategies

Using Integer Lattices (Torczon [1997], Audet and Dennis [2002])

- requires only simple decrease
- poll directions and step size must satisfy integer/rational requirements

Imposing Sufficient Decrease (Kolda, Lewis, and Torczon [2003])

- use of a forcing function
  $\rho : (0, +\infty) \to (0, +\infty)$, continuous, nondecreasing, and satisfying $\rho(t)/t \to 0$ when $t \downarrow 0$
- x is nondominated $\Leftrightarrow (F_X(x), h(x)) \notin D(L, \rho(\alpha))$



Convergence Results – Sequences

Refining sequence

A sequence $\{(x_k, \alpha_k)\}_{k\in K}$, such that $k \in K$ is an unsuccessful iteration and $\lim_{k\in K} \alpha_k = 0$.

Theorem (Refining Subsequences)

There is at least a convergent refining subsequence of iterates $\{x_k\}_{k\in K}$, corresponding to unsuccessful poll steps, with $\lim_{k\in K} \alpha_k = 0$.

Let $\overline{x}$ be the limit point of a convergent refining subsequence $\{x_k\}_{k\in K}$.

Definition (Refining Directions)

Refining directions for $\overline{x}$ are limit points of $\{d_k/\|d_k\|\}_{k\in K}$, where $d_k \in D_k$ and $x_k + \alpha_k d_k \in \mathcal{S} := \{x \in X \mid h(x) \leq h_{\max}\}$.

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Convergence Results

Proposition

Let $x \in L$ and y be a dominated point at an iteration associated with stepsize α. Then:

$$\exists j \in \{1, \ldots, m+1\} : f_j(y) > f_j(x) - \overline{\rho}(\alpha)$$

where:

- $\overline{\rho}(\cdot) \equiv 0$, if globalization is based on integer lattices
- $\overline{\rho}(\cdot) \equiv \rho(\cdot)$, if globalization is based on sufficient decrease

Convergence Results

Consider $\{x_k\}_{k \in K}$ a refining subsequence converging to $\overline{x} \in \mathcal{S} := \{x \in X \mid h(x) \leq h_{\texttt{max}}\}$. Assume that F and h are Lipschitz continuous near $\overline{x}$. Under any globalization strategy:

Theorem

- If $d \in \operatorname{int}(T_{\mathcal{S}}^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m+1\} \text{ such that } \quad f_j^{\circ}(\overline{x}; d) \geq 0$$

- If the set of refining directions for $\overline{x}$ is dense in $\operatorname{int}(T_{\mathcal{S}}^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point of $\overline{F}$ in $\mathcal{S}$:

$$\forall d \in T_{\mathcal{S}}^{Cl}(\overline{x}),\ \exists j = j(d) \in \{1, \ldots, m+1\} \ \text{ such that } \quad f_j^{\circ}(\overline{x}; d) \geq 0$$

Convergence Results – Infeasible case

Theorem

Consider $\{x_k\}_{k\in K}$ an **infeasible** refining subsequence such that for each $k \in K$, x_k is used at a **successful inexact restoration step**. Assume that h is continuous. Then DMS-FILTER-IR generates a **limit point** $\overline{x} \in \Upsilon$.

Convergence Results – Feasible case

Consider $\{x_k\}_{k\in K}$ a **feasible** refining subsequence converging to $\overline{x} \in \Upsilon$. Assuming a **globalization strategy based on integer latices**, we have:

Corollary

- If $d \in \operatorname{int}(T_{\Upsilon}^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m\} \text{ such that } \; f_j^{\circ}(\overline{x}; d) \geq 0$$

- If the set of refining directions for $\overline{x}$ is dense in $\operatorname{int}(T_{\Upsilon}^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point of F in Υ:

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Convergence Results – Feasible case

Consider $\{x_k\}_{k\in K}$ a **feasible** refining subsequence converging to $\overline{x} \in \Upsilon$. Assuming a **globalization strategy based on integer latices**, we have:

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Outline

Numerical Settings

- Comparison between:
  - **DMS-EB**: **Coordinate** versus **Dense**
  - **DMS-FILTER-IR**: **Coordinate** versus **Dense**
  - **DMS-EB** versus **DMS-FILTER-IR**: best version of each one
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of nonlinear constraints between 1 and 29
- Initialization:
  - **DMS-EB**: Feasible point provided by Karmitsa [2007]
  - **DMS-FILTER-IR**: n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - $\alpha_k < 10^{-3}$ for all points in the list
  - Maximum of 5000 function evaluations

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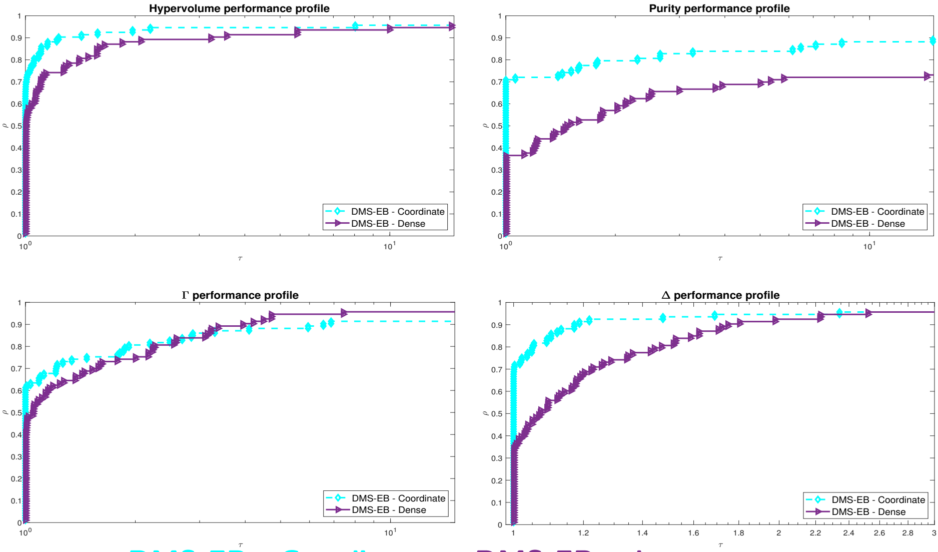
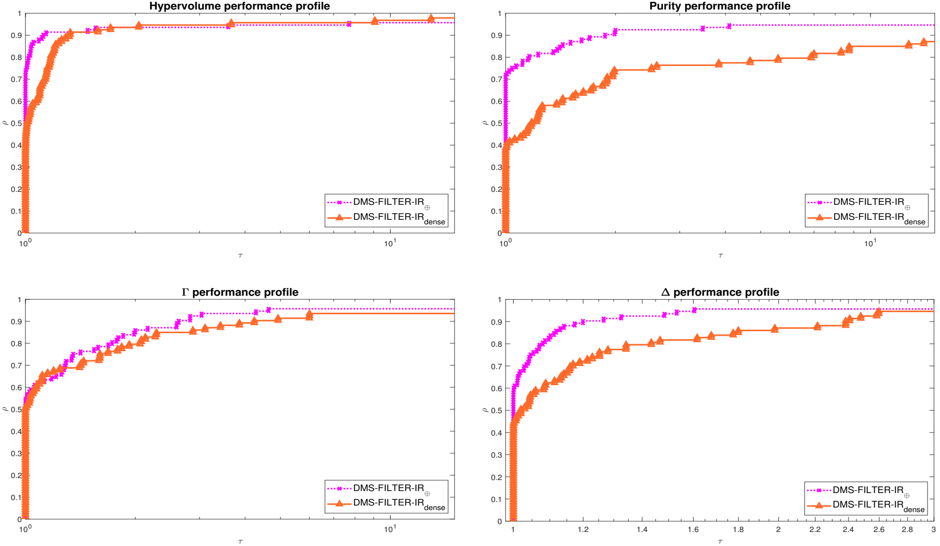
DMS - Coordinate vs Dense - 5k func. eval.



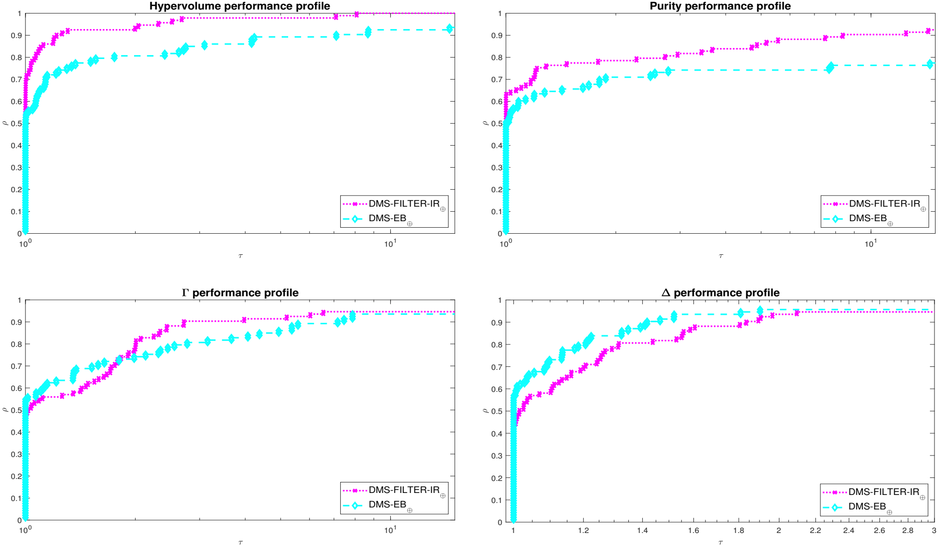
DMS-EB - Coordinate **DMS-EB - dense**

DMS-FILTER-IR - Coordinate vs Dense - 5k func. eval.



DMS-FILTER-IR - Coordinate **DMS-FILTER-IR - Dense**

Best version DMS vs DMS-FILTER-IR - 5k func. eval.



DMS-FILTER-IR - Coordinate **DMS-EB - Coordinate**

Numerical Settings

- Comparison among **DFMO**, **DMS-EB**, **DmultiMads-PB*** and **DMS-FILTER-IR**
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- Initialization:
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  - **Others:** n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS-EB** and **DMS-FILTER-IR**:
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  - **DFMO** and **DmultiMads-PB**:
    - default values
  - **All:** maximum of 500, 5000, and 20000 function evaluations

* **DmultiMads-PB** → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *Handling of constraints in multiobjective blackbox optimization*. ArXiv:2204.00904

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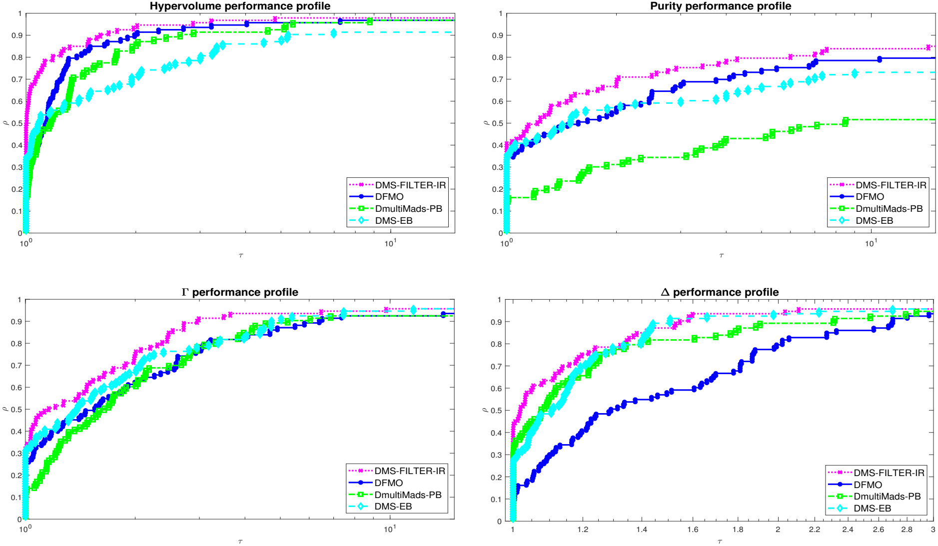
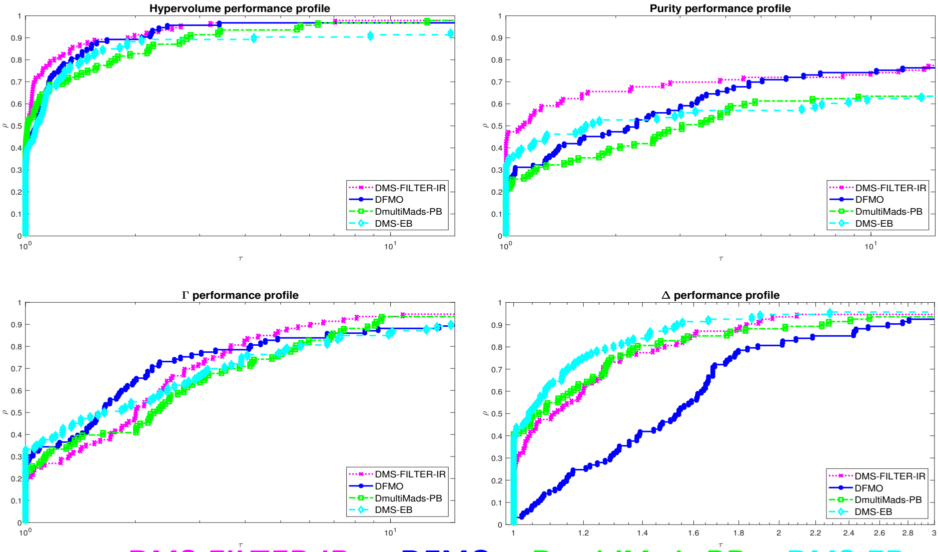
Results - 500 function evaluations



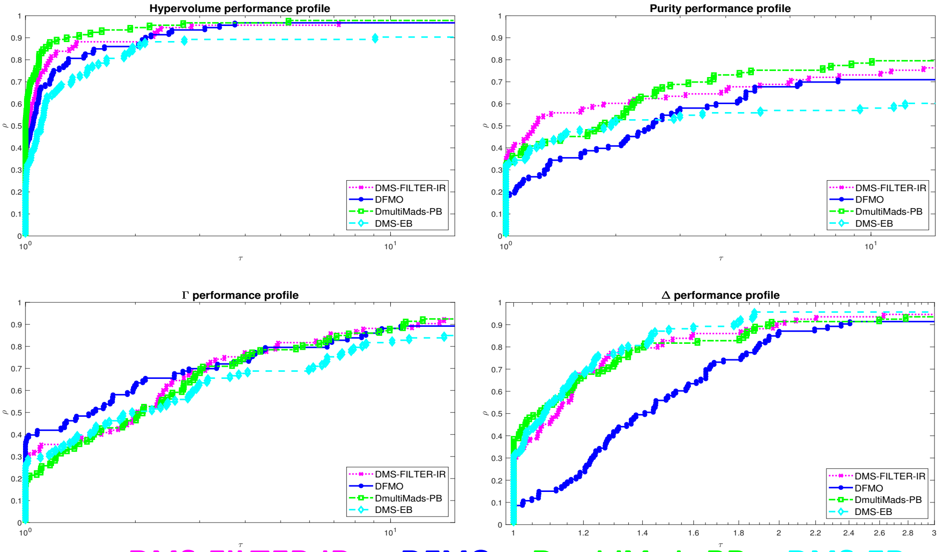
DMS-FILTER-IR **DFMO** **DmultiMads-PB** **DMS-EB**

Results - 5000 function evaluations



DMS-FILTER-IR **DFMO** **DmultiMads-PB** **DMS-EB**

Results - 20000 function evaluations



DMS-FILTER-IR DFMO DmultiMads-PB DMS-EB

Outline

Conclusions and Future Work

Conclusions

- DMS-FILTER-IR extends filter methods, with an inexact restoration step, to the DMS framework
- DMS-FILTER-IR presents a well-supported convergence analysis
- DMS-FILTER-IR presents competitive numerical results for constrained biobjective derivative-free optimization problems

Future Work

- developing a competitive numerical implementation for problems with more than two objectives

Thank you for your attention!

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