A Direct Multisearch Filter Method for Biobjective Optimization

Everton Jose da Silva
Advisor: Ana Luísa Custódio

Ph.D Student in Mathematics at Nova School of Science and Technology

UI/BD/151246/2021 PTDC/MAT-APL/28400/2017 UIDB/00297/2020 UIDP/00297/2020

Outline

Outline

Multiobjective Derivative-free Optimization

$$\min_{x \in \Omega \subseteq \mathbb{R}^n} \quad F(x) = \big(f_1(x), f_2(x), \ldots, f_m(x)\big)^\top$$

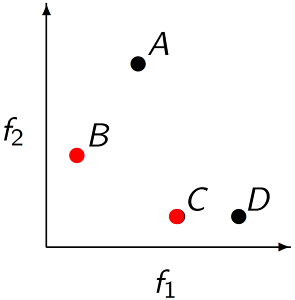
$$f_j : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}, \; j = 1, 2, \ldots, m \geq 2$$

- $\Omega = X \cap \{x \in \mathbb{R}^n \mid C(x) \leq 0\}$ where X is a full dimensional polyhedron and $C : \mathbb{R}^n \to (\mathbb{R} \cup \{+\infty\})^p$
- objectives often conflicting
- impossible to use or approximate derivatives
- expensive function evaluation

Make use of Pareto Dominance

Pareto Dominance (x dominates y)

$$F(x) \leq F(y), \quad \text{with } F(x) \neq F(y)$$



Pareto Front: $\{B, C\}$

Numerical Optimization

Iterative Methods

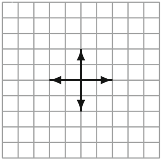
$$x_{k+1} = x_k + \alpha_k d_k$$

- Derivative-based methods: d_k should be a descent direction according to at least one of the objectives, i.e.

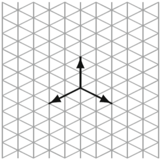
$$d_k^\top \nabla f_i(x_k) < 0, \qquad \text{with } i \in \{1, \ldots, m\}$$

- Derivative-free methods: when derivatives are not available and cannot be numerically approximated
  - Directional Direct Search: Uses positive spanning sets for sampling

    In $\mathbb{R}^2$:



$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$



$$\begin{bmatrix} \sqrt{3} & 0 & -\sqrt{3} \\ -1 & 2 & -1 \end{bmatrix}$$

$$pspan(D) = \mathbb{R}^2$$

Motivation

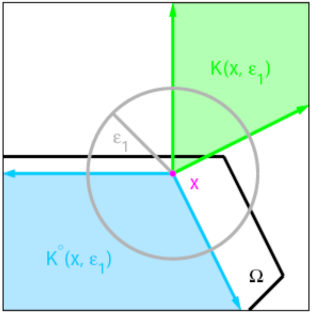
- DMS → A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente. *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109-1140
  - DMS_{dense} → Directions Asymptotically dense in the unit sphere
  - $\text{DMS}_{\oplus}$ → Coordinate directions

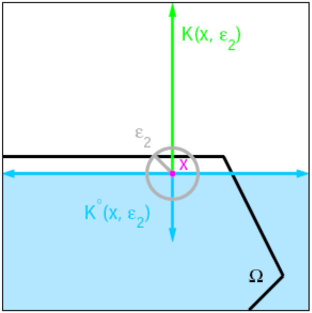
Motivation

- DMS → A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente. *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109-1140
  - DMS_{dense} → Directions Asymptotically dense in the unit sphere
  - $\text{DMS}_{\oplus}$ → Coordinate directions

DMS - General Linear Constraints

- Set of poll directions conforms to the geometry of the nearby constraints
- Approach of Abramson, Brezhneva, Dennis, and Pingel [2008] for single objective optimization





(in Kolda, Lewis, and Torczon [2003])

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

- Spreads Γ and Δ

$$\Gamma_{p,s} = \max_{j \in \{1,\ldots,m\}} \left(\max_{i \in \{0,\ldots,N\}} \{d_i\} \right)$$

$$\Delta = \max_{j \in \{1,\ldots,m\}} \left(\frac{d_0 + d_N + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_0 + d_N + (N-1)\bar{d}} \right)$$

- Hypervolume

$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

- Spreads Γ and Δ

$$\Gamma_{p,s} = \max_{j \in \{1,\ldots,m\}} \left(\max_{i \in \{0,\ldots,N\}} \{d_i\} \right)$$

$$\Delta = \max_{j \in \{1,\ldots,m\}} \left(\frac{d_0 + d_N + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_0 + d_N + (N-1)\bar{d}} \right)$$

- Hypervolume

$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

Metrics for Performance Profiles (Dolan and Moré [2002])

- Purity

$$\frac{|F_{p,s} \cap F_p|}{|F_{p,s}|}$$

- Spreads Γ and Δ

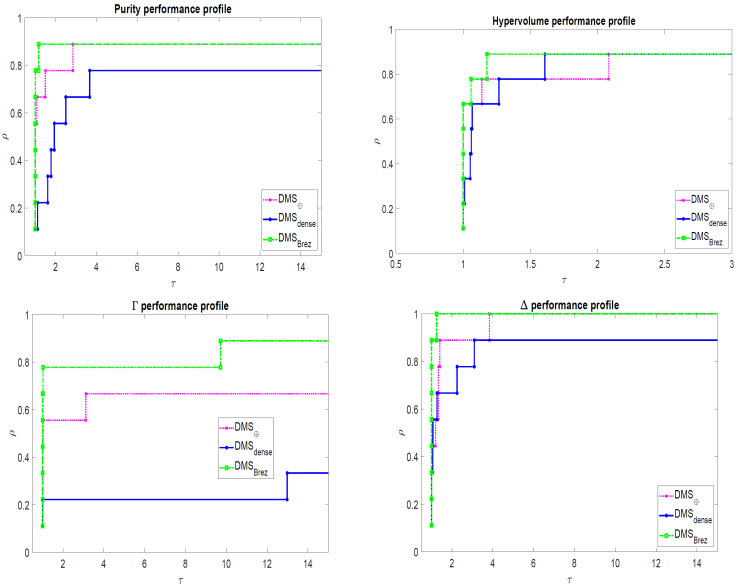
$$\Gamma_{p,s} = \max_{j \in \{1,\dots,m\}} \left(\max_{i \in \{0,\dots,N\}} \{d_i\} \right)$$

$$\Delta = \max_{j \in \{1,\dots,m\}} \left(\frac{d_0 + d_N + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_0 + d_N + (N-1)\bar{d}} \right)$$

- Hypervolume

$$HI_{p,s} = Vol\{b \in \mathbb{R}^m \mid b \leq U_p \wedge \exists a \in F_{p,s} : a \leq b\}$$

DMS - General Linear Constraints



Motivation

- DMS → A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente. *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109-1140
- DFMO → G. Liuzzi, S. Lucidi, and F. Rinaldi. *A derivative-free approach to constrained multiobjective nonsmooth optimization*. SIAM J. Optim. (2016), 26, 2744-2774
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29

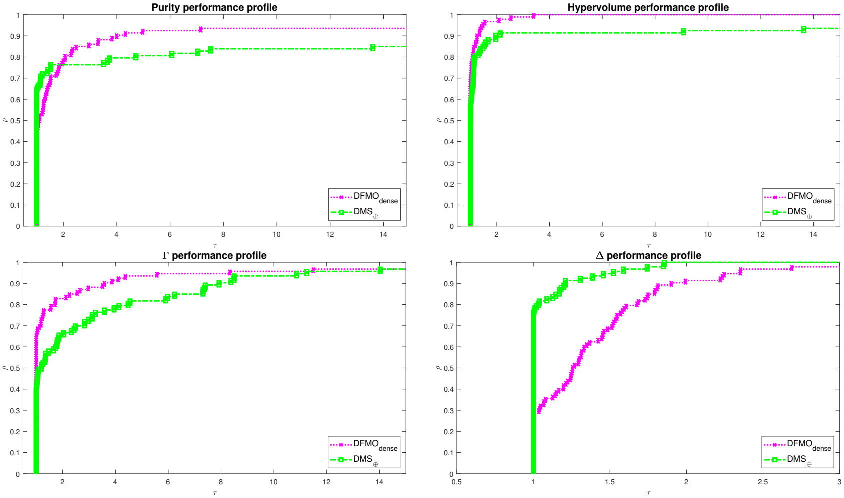
Motivation

- DMS → A. L. Custódio, J. F. A. Madeira, A. I. F. Vaz, and L. N. Vicente. *Direct multisearch for multiobjective optimization*, SIAM J. Optim. (2011), 21, 1109-1140

- DFMO → G. Liuzzi, S. Lucidi, and F. Rinaldi. *A derivative-free approach to constrained multiobjective nonsmooth optimization.* SIAM J. Optim. (2016), 26, 2744-2774

- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29

DMS - Nonlinear + Bound Constraints



-DFMO

-DMS

New Problem

- Extreme Barrier Function:

$$F_X(x) = \begin{cases} F(x), \text{ if } x \in X \\ (+\infty, +\infty, \dots, +\infty)^\top, \text{ otherwise} \end{cases}$$

- Constraint Violation function:

$$h(x) = \|C(x)_+\|_2^2 = \sum_{i=1}^{p} \max\{0, c_i(x)\}^2$$

$$\min_{x \in X} \left(f_1(x), f_2(x), \dots, f_m(x), h(x) \right)^\top$$

New Problem

- Extreme Barrier Function:

$$F_X(x) = \begin{cases} F(x), \text{ if } x \in X \\ (+\infty, +\infty, \ldots, +\infty)^\top, \text{ otherwise} \end{cases}$$

- Constraint Violation function:

$$h(x) = \|C(x)_+\|_2^2 = \sum_{i=1}^{p} \max\{0, c_i(x)\}^2$$

$$\min_{x \in X} \left(f_1(x), f_2(x), \ldots, f_m(x), h(x)\right)^\top$$

New Problem

- Extreme Barrier Function:

$$F_X(x) = \begin{cases} F(x), \text{ if } x \in X \\ (+\infty, +\infty, \dots, +\infty)^\top, \text{ otherwise} \end{cases}$$

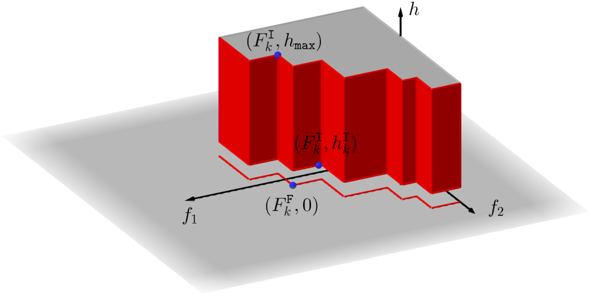
- Constraint Violation function:

$$h(x) = \|C(x)_+\|_2^2 = \sum_{i=1}^{p} \max\{0, c_i(x)\}^2$$

$$\min_{x \in X} \ \left(f_1(x), f_2(x), \dots, f_m(x), h(x)\right)^\top$$

Filter Approach

The filter $\mathcal{F}$ is a set of nondominated points

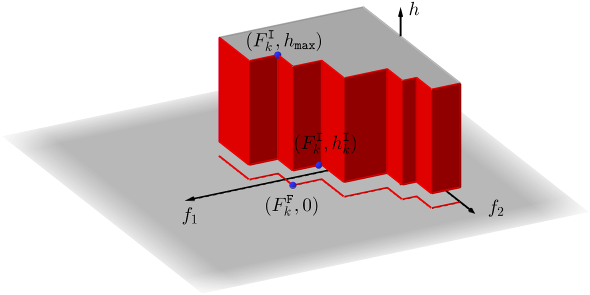


A point x' is said to be filtered by a filter $\mathcal{F}$ if any of the following properties hold:

- There exists a point $x \in \mathcal{F}$ such that $x' \geq x$
- $h(x') > h_{\max}$ for some positive finite upper bound $h_{\max}$

Filter Approach

The filter $\mathcal{F}$ is a set of nondominated points

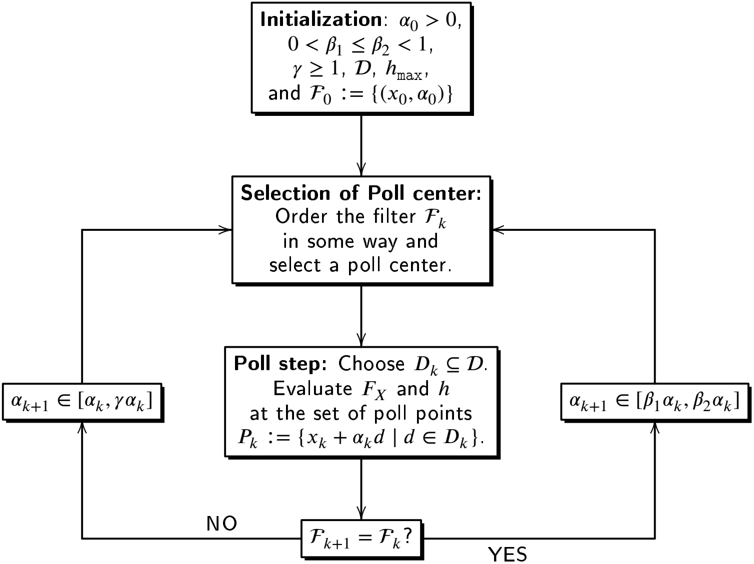


A point x' is said to be filtered by a filter $\mathcal{F}$ if any of the following properties hold:

- There exists a point $x \in \mathcal{F}$ such that $x' \succeq x$
- $h(x') > h_{\mathtt{max}}$ for some positive finite upper bound $h_{\mathtt{max}}$

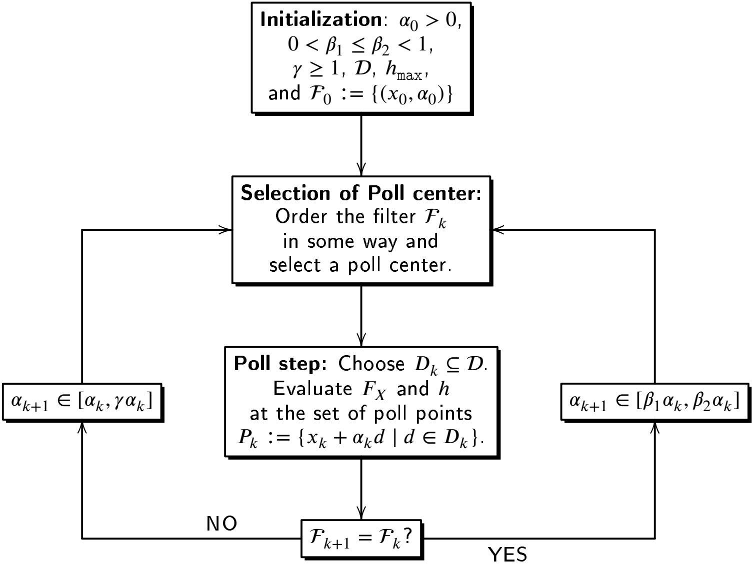
Outline

DMS-Filter - Algorithmic Structure



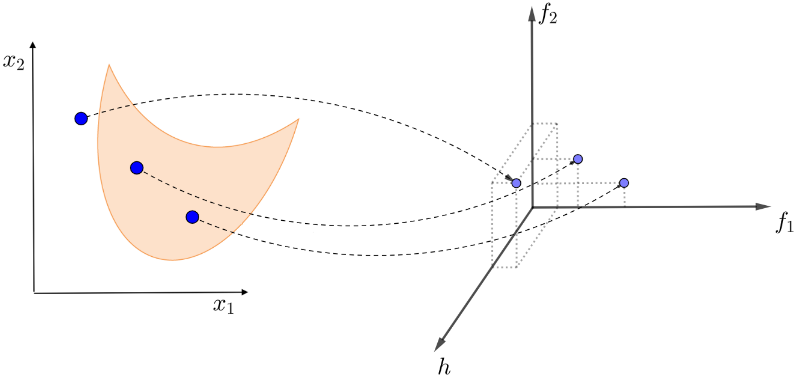
Solutions: $L := \{(x, \alpha) \in \mathcal{F} \mid (F_X(x), h(x)) = (F(x), 0)\}$.

DMS-Filter - Algorithmic Structure

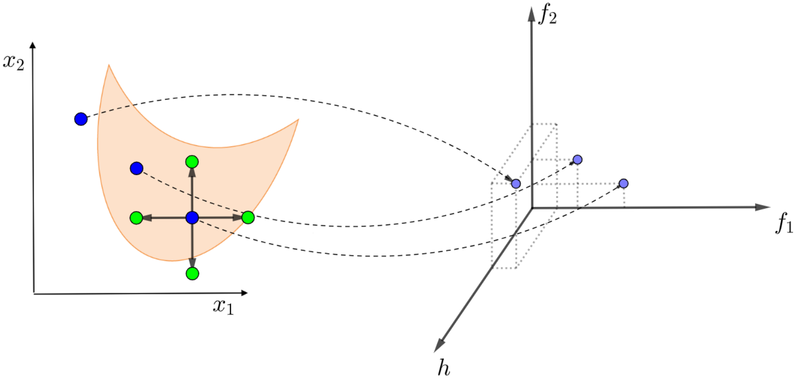


Solutions: $L := \{(x, \alpha) \in \mathcal{F} \mid (F_X(x), h(x)) = (F(x), 0)\}$.

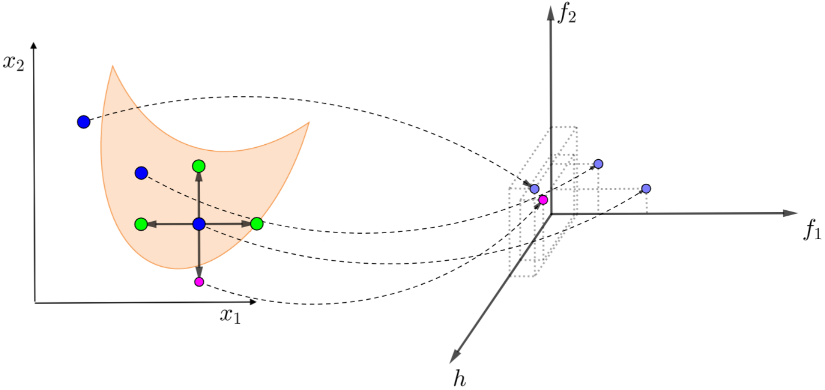
DMS-Filter – Poll Step



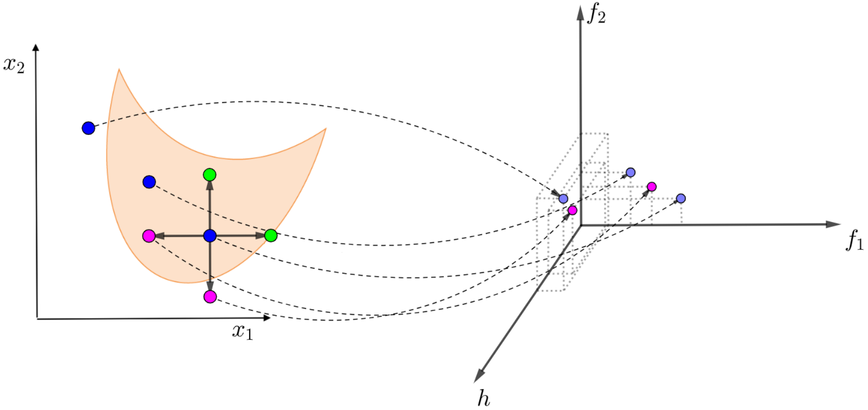
DMS-Filter – Poll Step



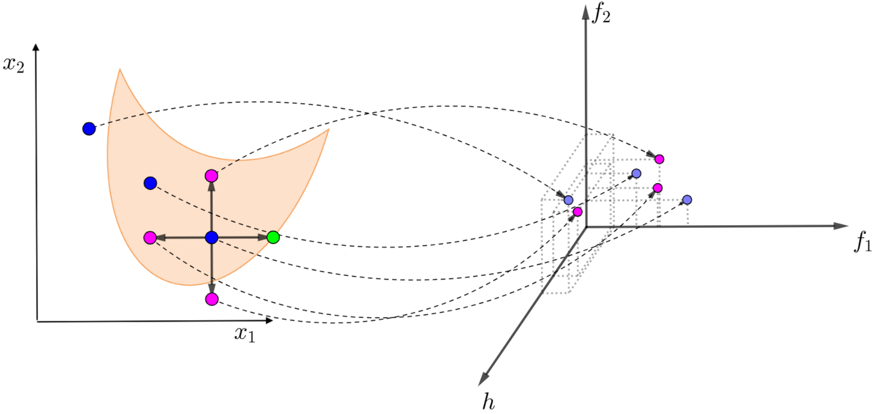
DMS-Filter – Poll Step



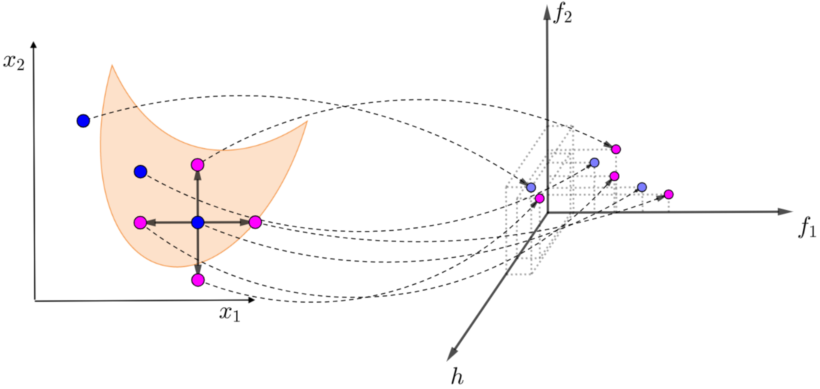
DMS-Filter – Poll Step



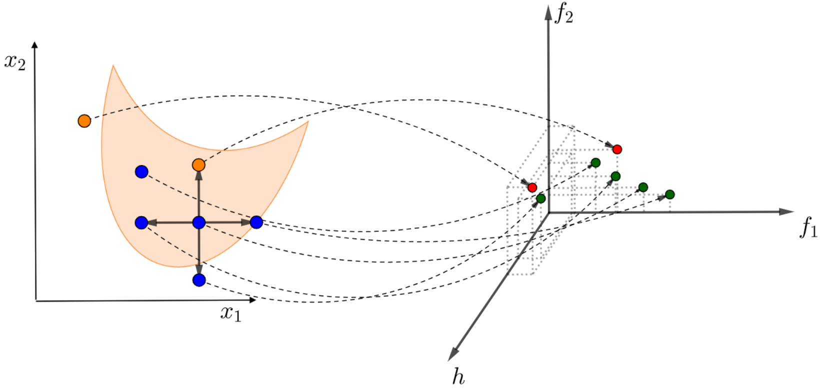
DMS-Filter – Poll Step



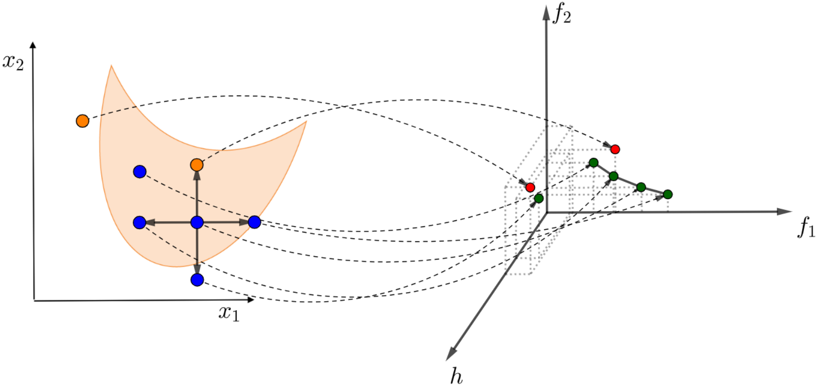
DMS-Filter – Poll Step



DMS-Filter – Poll Step

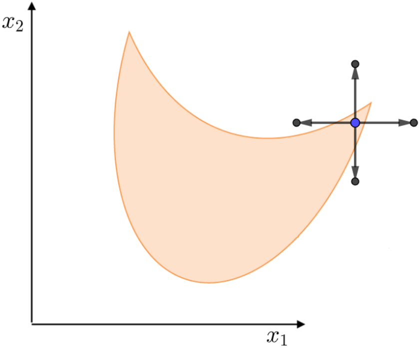


DMS-Filter – Poll Step

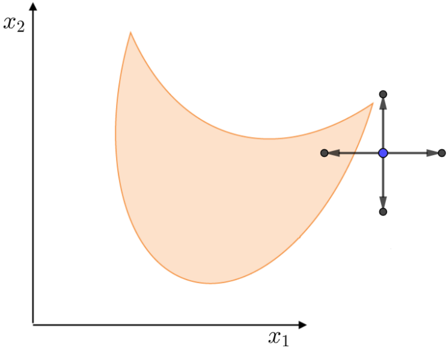


Poll Center Selection

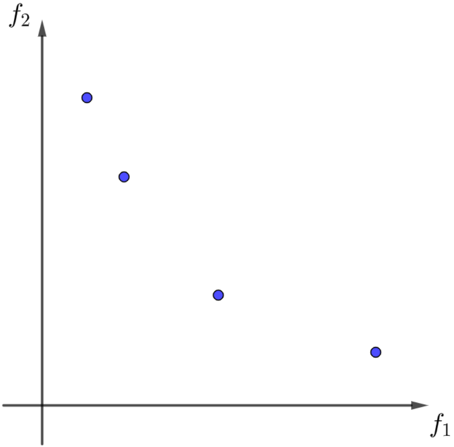
- **Feasible to Infeasible**



- **Infeasible to Feasible**

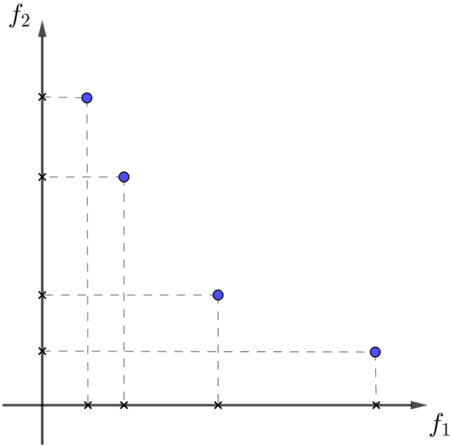


Feasible poll center - Most Isolated Point



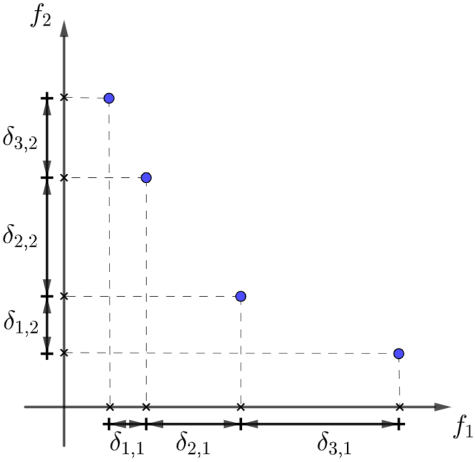
- $\mathcal{F} \cap \Omega$

Feasible poll center - Most Isolated Point



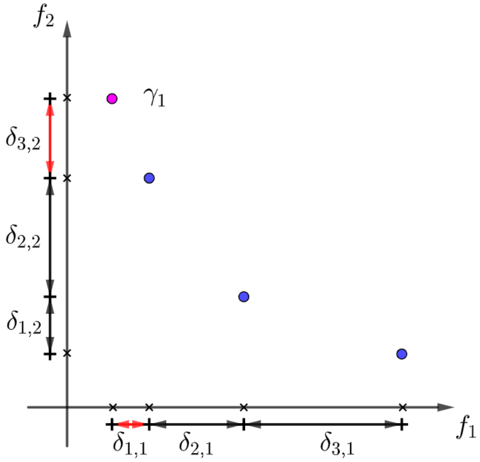
- $\mathcal{F} \cap \Omega$

Feasible poll center - Most Isolated Point



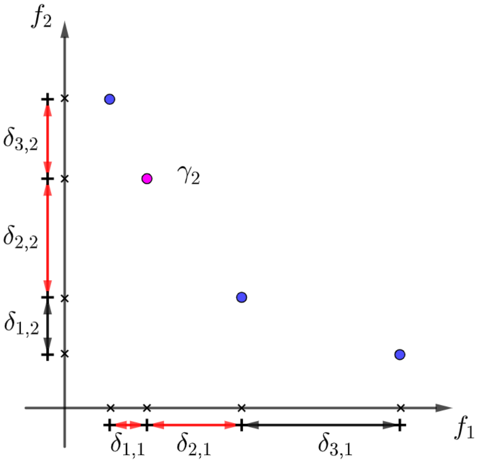
$\delta_{i,j} = f_{i+1,j} - f_{i,j}$
for $i = 1, 2, 3$ and $j = 1, 2$.

Feasible poll center - Most Isolated Point



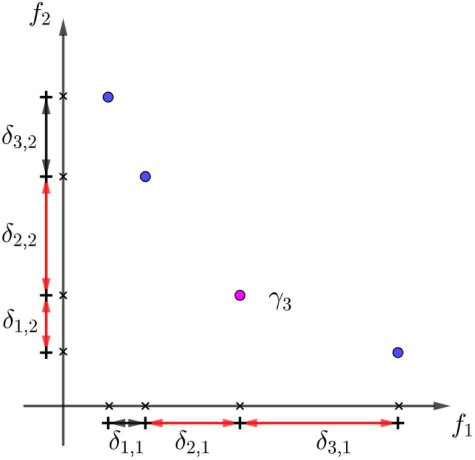
$$\gamma_1 = \frac{\delta_{3,2} + \delta_{1,1}}{2}$$

Feasible poll center - Most Isolated Point



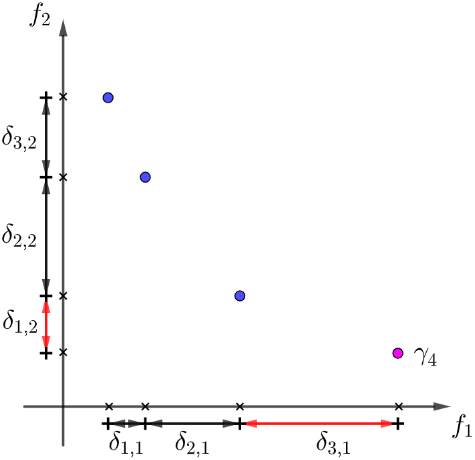
$$\gamma_2 = \frac{\frac{\delta_{3,2}+\delta_{2,2}}{2} + \frac{\delta_{1,1}+\delta_{2,1}}{2}}{2}$$

Feasible poll center - Most Isolated Point



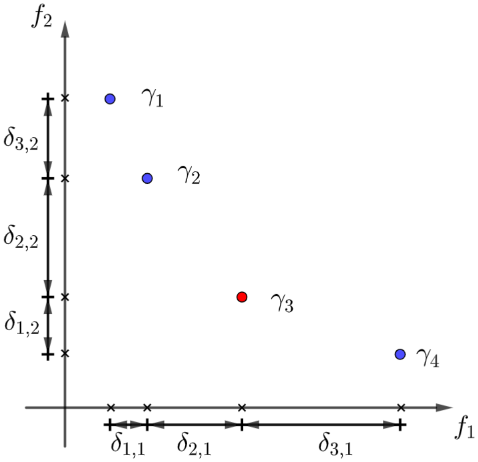
$$\gamma_3 = \frac{\frac{\delta_{2,2}+\delta_{1,2}}{2} + \frac{\delta_{2,1}+\delta_{3,1}}{2}}{2}$$

Feasible poll center - Most Isolated Point



$$\gamma_4 = \frac{\delta_{1,2} + \delta_{3,1}}{2}$$

Feasible poll center - Most Isolated Point



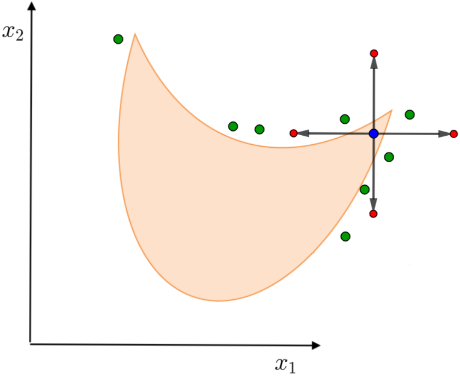
$$\gamma_1 = \frac{\delta_{3,2} + \delta_{1,1}}{2}$$

$$\gamma_2 = \frac{\frac{\delta_{3,2}+\delta_{2,2}}{2} + \frac{\delta_{1,1}+\delta_{2,1}}{2}}{2}$$

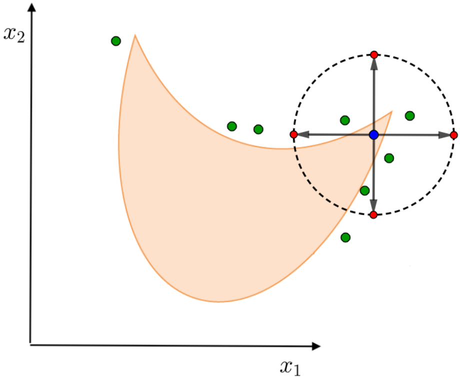
$$\gamma_3 = \frac{\frac{\delta_{2,2}+\delta_{1,2}}{2} + \frac{\delta_{2,1}+\delta_{3,1}}{2}}{2}$$

$$\gamma_4 = \frac{\delta_{1,2} + \delta_{3,1}}{2}$$

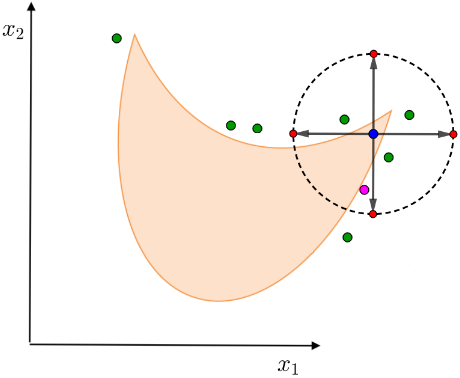
Infeasible poll center



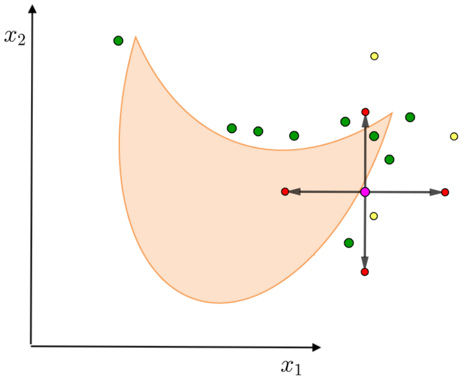
Infeasible poll center



Infeasible poll center



Infeasible poll center



Outline

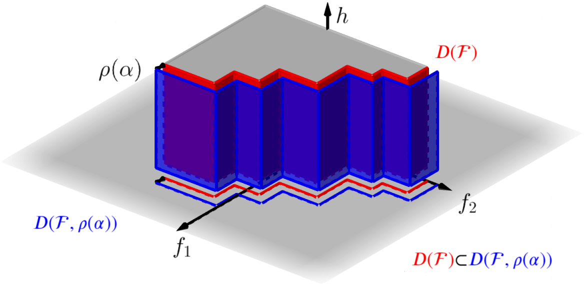
Globalization Strategies

Using Integer Lattices (Torczon [1997], Audet and Dennis [2002])

- requires only simple decrease
- poll directions and step size must satisfy integer/rational requirements

Imposing Sufficient Decrease (Kolda, Lewis, and Torczon [2003])

- use of a forcing function
  $\rho : (0, +\infty) \to (0, +\infty)$, continuous and nondecreasing, satisfying
  $\rho(t)/t \to 0$ when $t \downarrow 0$
- x is nondominated $\Leftrightarrow (F_X(x), h(x)) \notin D(\mathcal{F}, \rho(\alpha))$



Convergence Results

Theorem (Refining Subsequences)

There is at least a convergent refining subsequence of iterates $\{x_k\}_{k\in K}$, corresponding to unsuccessful poll steps, with $\lim_{k\in K} \alpha_k = 0$.

Let $\overline{x}$ be the limit point of a convergent refining subsequence $\{x_k\}_{k\in K}$.

Definition (Refining Directions)

Refining directions for $\overline{x}$ are limit points of $\{d_k/\|d_k\|\}_{k\in K}$, where $d_k \in D_k$ and $x_k + \alpha_k d_k \in S := \{x \in X \mid h(x) \le h_{\max}\}$.

Convergence Results

Theorem (Refining Subsequences)

There is at least a convergent refining subsequence of iterates $\{x_k\}_{k \in K}$, corresponding to unsuccessful poll steps, with $\lim\limits_{k \in K} \alpha_k = 0$.

Let $\bar{x}$ be the limit point of a convergent refining subsequence $\{x_k\}_{k \in K}$.

Definition (Refining Directions)

Refining directions for $\bar{x}$ are limit points of $\{d_k / \|d_k\|\}_{k \in K}$, where $d_k \in D_k$ and $x_k + \alpha_k d_k \in S := \{x \in X \mid h(x) \leq h_{\max}\}$.

Some Definitions

Clarke Tangent Cone

$$T_S^{Cl}(x) := \{d \in \mathbb{R}^n \mid \forall \{y_k\} \in S,\ y_k \to x,\ \forall \{t_k\} \in \mathbb{R}_+,\ t_k \downarrow 0,\ \exists \{w_k\} \in \mathbb{R}^n,$$

$$\mathrm{w}_k \to d, \ \text{ such that } y_k + t_k w_k \in S\}.$$

Clarke-Jahn Generalized Derivative

Let $g : \mathbb{R}^n \to \mathbb{R}$ be Lipschitz continuous near $\overline{x} \in \mathbb{R}^n$ we can define the Clarke-Jahn generalized derivatives of g along d in the $int(T_S^{Cl}(x))$ to $S \subset \mathbb{R}^n$ at x,

$$g^\circ(x; d) := \limsup_{\substack{x' \to x, x' \in S \\ t \downarrow 0, x' + td \in S}} \frac{g(x' + td) - g(x')}{t}.$$

Some Definitions

Clarke Tangent Cone

$$T_S^{Cl}(x) := \{d \in \mathbb{R}^n \mid \forall\{y_k\} \in S,\ y_k \to x,\ \forall\{t_k\} \in \mathbb{R}_+,\ t_k \downarrow 0,\ \exists\{w_k\} \in \mathbb{R}^n,$$

$$\mathrm{w}_k \to d, \ \text{ such that } y_k + t_k w_k \in S\}.$$

Clarke-Jahn Generalized Derivative

Let $g : \mathbb{R}^n \to \mathbb{R}$ be Lipschitz continuous near $\bar{x} \in \mathbb{R}^n$ we can define the Clarke-Jahn generalized derivatives of g along d in the $int(T_S^{Cl}(x))$ to $S \subset \mathbb{R}^n$ at x,

$$g^\circ(x;d) := \limsup_{\substack{x' \to x, x' \in S \\ t \downarrow 0, x' + td \in S}} \frac{g(x'+td) - g(x')}{t}.$$

Convergence Results

Assume that F and h are Lipschitz continuous near $\overline{x}$.

Theorem

- Let $\{x_k^{\mathrm{I}}\}_{k\in K}$ be an infeasible refining subsequence converging to $\overline{x} \in S$. If $d \in \mathrm{int}(T_S^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$h^{\circ}(\overline{x}; d) \geq 0$$

- Let $\{x_k^{\mathrm{F}}\}_{k\in K}$ be a feasible refining subsequence converging to $\overline{x} \in \Omega$. If $d \in \mathrm{int}(T_\Omega^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

Convergence Results

Assume that F and h are Lipschitz continuous near $\overline{x}$.

Theorem

- Let $\{x_k^{\mathrm{I}}\}_{k\in K}$ be an infeasible refining subsequence converging to $\overline{x} \in \mathcal{S}$. If $d \in \operatorname{int}(T_{\mathcal{S}}^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$h^{\circ}(\overline{x}; d) \geq 0$$

- Let $\{x_k^{\mathrm{F}}\}_{k\in K}$ be a feasible refining subsequence converging to $\overline{x} \in \Omega$. If $d \in \operatorname{int}(T_{\Omega}^{Cl}(\overline{x}))$ is a refining direction for $\overline{x}$ then:

$$\exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

Convergence Results

Theorem

- Let $\{x_k^{\mathrm{I}}\}_{k\in K}$ be an infeasible refining subsequence converging to $\overline{x} \in S$. If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_S^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Clarke critical point:

$$\forall d \in T_S^{Cl}(\overline{x}), h^{\circ}(\overline{x}; d) \geq 0$$

- Let $\{x_k^{\mathrm{F}}\}_{k\in K}$ be a feasible refining subsequence converging to $\overline{x} \in \Omega$. If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_\Omega^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point:

$$\forall d \in T_\Omega^{Cl}(\overline{x}), \exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

Convergence Results

Theorem

- Let $\{x_k^{\mathrm{I}}\}_{k\in K}$ be an infeasible refining subsequence converging to $\overline{x} \in S$. If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_S^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Clarke critical point:

$$\forall d \in T_S^{Cl}(\overline{x}), h^{\circ}(\overline{x}; d) \geq 0$$

- Let $\{x_k^{\mathrm{F}}\}_{k\in K}$ be a feasible refining subsequence converging to $\overline{x} \in \Omega$. If the set of refining directions for $\overline{x}$ is dense in $\mathrm{int}(T_\Omega^{Cl}(\overline{x})) \neq \emptyset$ then $\overline{x}$ is a Pareto-Clarke critical point:

$$\forall d \in T_\Omega^{Cl}(\overline{x}), \exists j = j(d) \in \{1, \ldots, m\} \text{ such that } f_j^{\circ}(\overline{x}; d) \geq 0$$

Outline

Numerical Settings

- Comparison among DFMO, DMS and DMS-Filter
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization with a feasible point
  - Feasible point provided by Karmitsa [2007]
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS** and **DMS-Filter**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO**:
    - default values
  - maximum of 20000 function evaluations

Numerical Settings

- Comparison among DFMO, DMS and DMS-Filter
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization with a feasible point
  - Feasible point provided by Karmitsa [2007]
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS** and **DMS-Filter**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO**:
    - default values
  - maximum of 20000 function evaluations

Numerical Settings

- Comparison among DFMO, DMS and DMS-Filter
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization with a feasible point
  - Feasible point provided by Karmitsa [2007]
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS** and **DMS-Filter**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO**:
    - default values
  - maximum of 20000 function evaluations

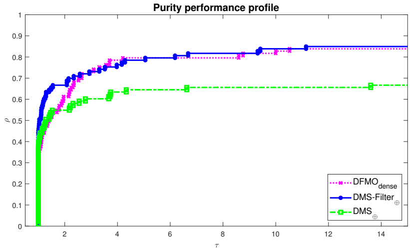
Numerical Settings

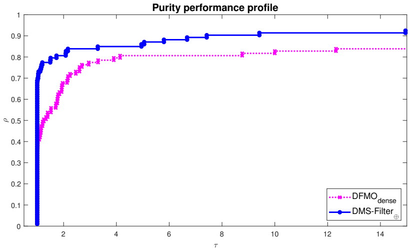
- Comparison among DFMO, DMS and DMS-Filter
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization with a feasible point
  - Feasible point provided by Karmitsa [2007]
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS** and **DMS-Filter**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO**:
    - default values
  - maximum of 20000 function evaluations

Numerical Settings

- Comparison among DFMO, DMS and DMS-Filter
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization with a feasible point
  - Feasible point provided by Karmitsa [2007]
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criterion
  - **DMS** and **DMS-Filter**:
    - $\alpha_k < 10^{-3}$ for all points in the filter
  - **DFMO**:
    - default values
  - maximum of 20000 function evaluations

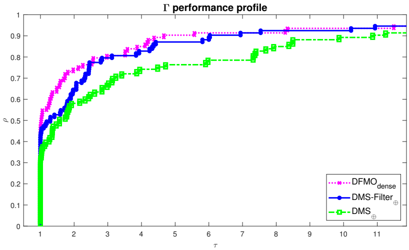
Results - Purity

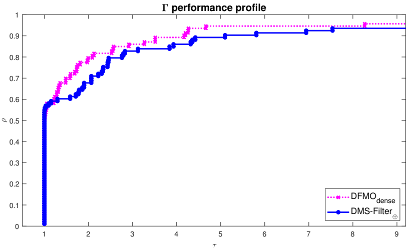




- DFMO
- DMS-Filter
- DMS

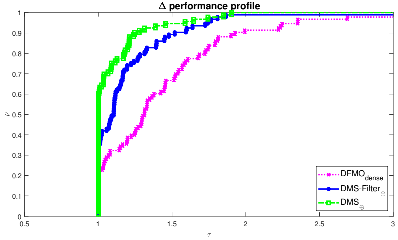
Results - Spread Gamma (Γ)

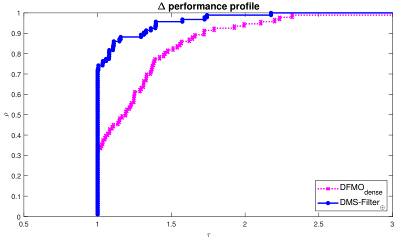




- DFMO
- DMS-Filter
- DMS

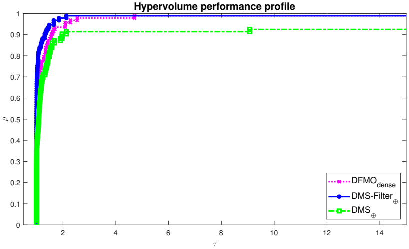
Results - Spread Delta (Δ)

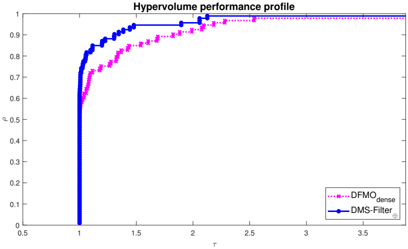




- DFMO
- DMS-Filter
- DMS

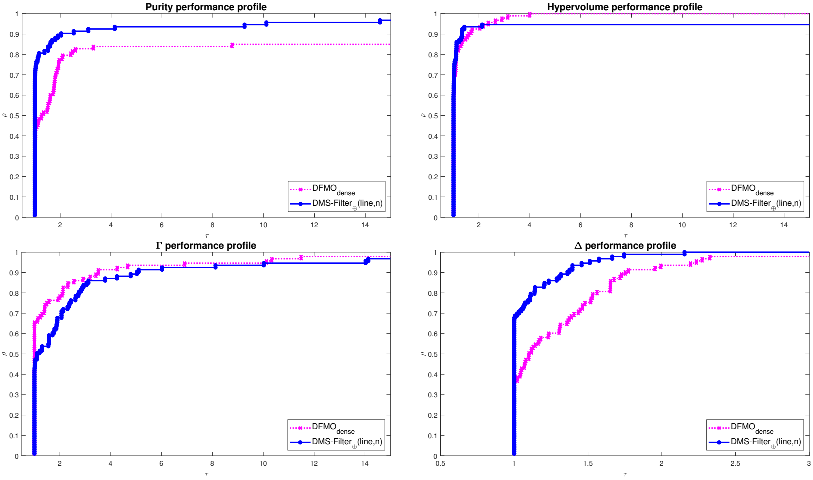
Results - Hypervolume





- DFMO
- DMS-Filter
- DMS

Results - DMS-Filter(line,n) VS DFMO

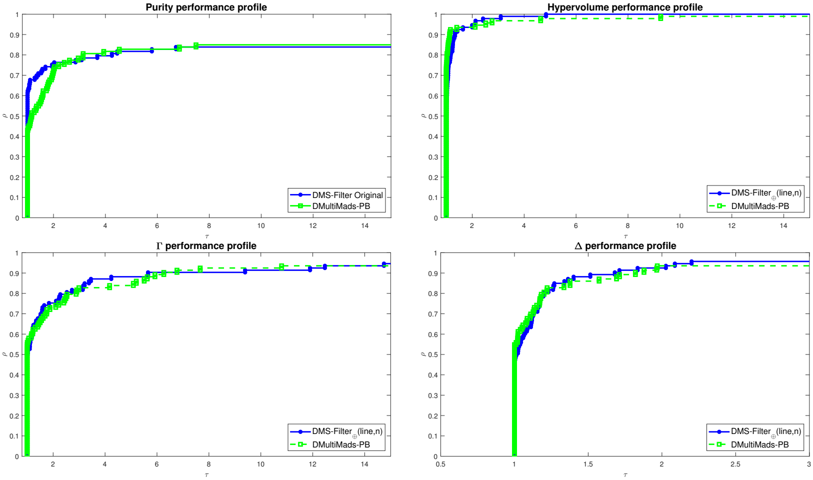


- DFMO
- DMS-Filter

Numerical Settings

- Comparison between DMS-Filter and DMultiMads-PB
  - **DMultiMads-PB** $\rightarrow$ Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *Handling of constraints in multiobjective blackbox optimization*. ArXiv:2204.00904
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criteria - default and allowing a maximum of 20000 function evaluations for both solvers

DMS-Filter VS DMultiMads-PB



DMS-Filter **DMultiMads-PB**

Question

Are we starting from a strong code, or already in the basic version the code is not competitive?

- Problems:
  - 100 bound constrained MOO problems
  - number of variables between 1 and 30
  - number of objectives between 2 and 4
- Solvers:
  - DMS
  - DMultiMads-EB → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *DMulti-MADS: mesh adaptive direct multisearch for bound-constrained blackbox multiobjective optimization.* Computational Optimization and Applications, Springer Verlag, 2021, 79 (2), pp.301-338.
  - MOIF → G. Cocchi, G. Liuzzi, A. Papini and M. Sciandrone. *An implicit filtering algorithm for derivative-free multiobjective optimization with box constraints*, Comput Optim Appl (2018) 69:267–296.
- Stopping criteria - one $\alpha_k < 10^{-9}$ and maximum of 20000 function evaluations

Question

Are we starting from a strong code, or already in the basic version the code is not competitive?

- Problems:
  - 100 bound constrained MOO problems
  - number of variables between 1 and 30
  - number of objectives between 2 and 4
- Solvers:
  - DMS
  - DMultiMads-EB → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *DMulti-MADS: mesh adaptive direct multisearch for bound-constrained blackbox multiobjective optimization.* Computational Optimization and Applications, Springer Verlag, 2021, 79 (2), pp.301-338.
  - MOIF → G. Cocchi, G. Liuzzi, A. Papini and M. Sciandrone. *An implicit filtering algorithm for derivative-free multiobjective optimization with box constraints*, Comput Optim Appl (2018) 69:267–296.
- Stopping criteria - one $\alpha_k < 10^{-9}$ and maximum of 20000 function evaluations

Question

Are we starting from a strong code, or already in the basic version the code is not competitive?

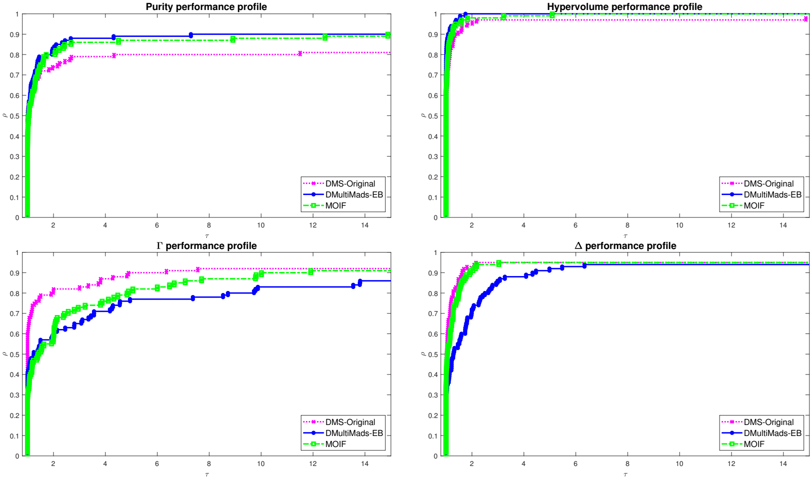
- Problems:
  - 100 bound constrained MOO problems
  - number of variables between 1 and 30
  - number of objectives between 2 and 4
- Solvers:
  - DMS
  - DMultiMads-EB → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *DMulti-MADS: mesh adaptive direct multisearch for bound-constrained blackbox multiobjective optimization*. Computational Optimization and Applications, Springer Verlag, 2021, 79 (2), pp.301-338.
  - MOIF → G. Cocchi, G. Liuzzi, A. Papini and M. Sciandrone. *An implicit filtering algorithm for derivative-free multiobjective optimization with box constraints*, Comput Optim Appl (2018) 69:267–296.
- Stopping criteria - one $\alpha_k < 10^{-9}$ and maximum of 20000 function evaluations

Question

Are we starting from a strong code, or already in the basic version the code is not competitive?

- Problems:
  - 100 bound constrained MOO problems
  - number of variables between 1 and 30
  - number of objectives between 2 and 4
- Solvers:
  - DMS
  - DMultiMads-EB → Jean Bigeon, Sébastien Le Digabel, Ludovic Salomon. *DMulti-MADS: mesh adaptive direct multisearch for bound-constrained blackbox multiobjective optimization*. Computational Optimization and Applications, Springer Verlag, 2021, 79 (2), pp.301-338.
  - MOIF → G. Cocchi, G. Liuzzi, A. Papini and M. Sciandrone. *An implicit filtering algorithm for derivative-free multiobjective optimization with box constraints*, Comput Optim Appl (2018) 69:267–296.
- Stopping criteria - one $\alpha_k < 10^{-9}$ and maximum of 20000 function evaluations

DMS original versus MOIF and DMultiMads-EB



- DMS-Original
- MOIF
- DMultiMads-EB

Improvements in DMS

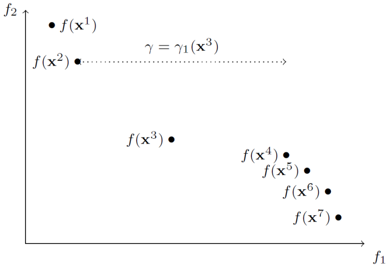
- Selection of poll center based on DMultiMads

$$L^{\texttt{select}} := \left\{ (x,\alpha) \in L^k \mid \alpha \geq \tau^{\omega^+} \alpha^k_{\max} \right\}$$

with $\alpha^k_{\max} = \max_{j=1,2,\ldots,|L^k|} \alpha^j$, $\tau \in (0,1)$ and $\omega^+ \in \mathbb{N}$

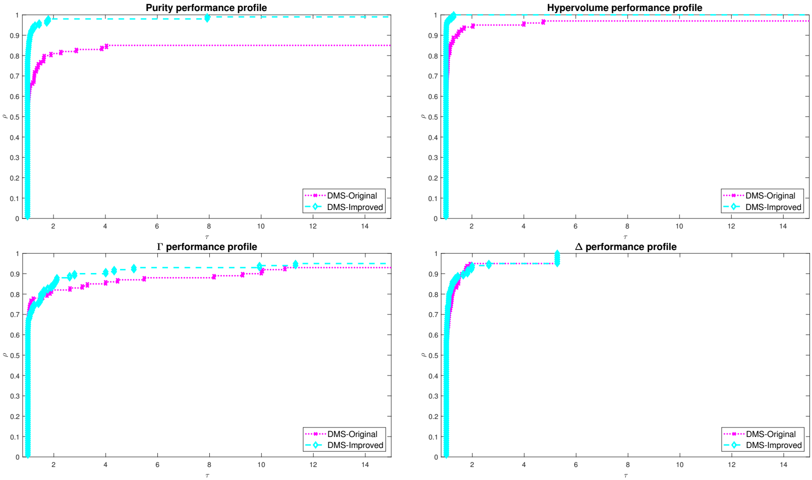
- Gamma Γ

$$\gamma_i\left(\mathbf{x}^j\right) = \begin{cases} 2\dfrac{f_i\left(\mathbf{x}^2\right) - f_i\left(\mathbf{x}^1\right)}{f_i\left(\mathbf{x}^{|L^k|}\right) - f_i\left(\mathbf{x}^1\right)} & \text{if } j = 1 \\ 2\dfrac{f_i\left(\mathbf{x}^{|L^k|}\right) - f_i\left(\mathbf{x}^{|L^k|-1}\right)}{f_i\left(\mathbf{x}^{|L^k|}\right) - f_i\left(\mathbf{x}^1\right)} & \text{if } j = |L^k| \\ \dfrac{f_i\left(\mathbf{x}^{j+1}\right) - f_i\left(\mathbf{x}^{j-1}\right)}{f_i\left(\mathbf{x}^{|L^k|}\right) - f_i\left(\mathbf{x}^1\right)} & \text{otherwise.} \end{cases}$$



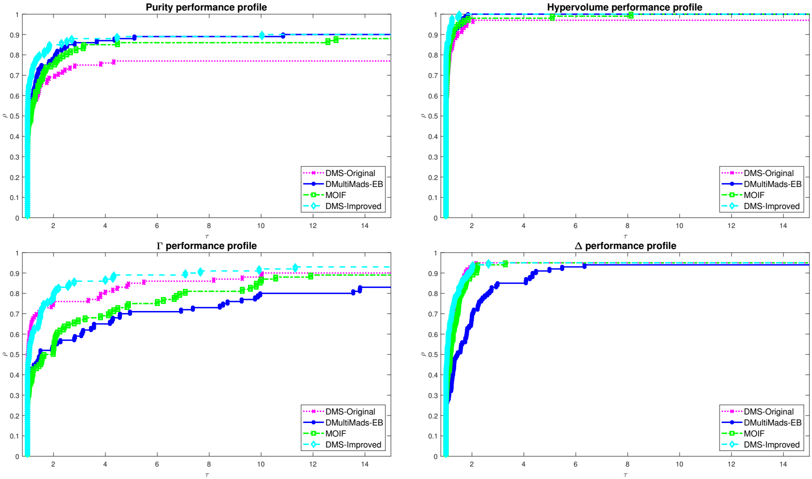
$$\gamma = \max_{j=1,\ldots,|L^{\texttt{select}}|} \max_{i=1,\ldots,p} \gamma_i(x^j)$$

Improvements in DMS



- DMS-Original
- DMS-Improved - $\omega^+ = 6$

DMS-Improved versus MOIF and DMultiMads-EB



- DMS-Original
- DMultiMads-EB
- MOIF
- DMS-Improved - $\omega^{+} = 6$

DMS-Filter Improvements

- If the poll center chosen is infeasible, we update the parameter $h^k_{\mathtt{max}}$

$$h^{k+1}_{\max} := \begin{cases} \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^t\right) < h\left(x^k_I\right) \right\} & \text{if iter } k \text{ is improving,} \\ h\left(x^k_I\right) & \text{if } h\left(x^k_I\right) = \max\limits_{x \in I^k} h(x), \\ \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^k_I\right) \le h\left(x^t\right) < \max\limits_{x \in I^k} h(x) \right\} & \text{otherwise.} \end{cases}$$

  - Generates a nonincreasing sequence of parameters such that $h^k_{\mathtt{max}} \to 0$

- Calibration of parameter ω^+: $\omega^+ = 0$

$$L^{\mathtt{select}} := \left\{ (x, \alpha) \in \mathcal{F}^k \mid h(x) = 0 \ \text{ and } \alpha = \alpha^k_{\max} \right\}$$

with $\alpha^k_{\max} = \max_{j=1,2,\ldots,|\mathcal{F}^k|} \alpha^j$

DMS-Filter Improvements

- If the poll center chosen is infeasible, we update the parameter $h^k_{\mathtt{max}}$

$$h^{k+1}_{\max} := \begin{cases} \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^t\right) < h\left(x^k_I\right) \right\} & \text{if iter } k \text{ is improving,} \\ h\left(x^k_I\right) & \text{if } h\left(x^k_I\right) = \max\limits_{x \in I^k} h(x), \\ \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^k_I\right) \leq h\left(x^t\right) < \max\limits_{x \in I^k} h(x) \right\} & \text{otherwise.} \end{cases}$$

  - Generates a nonincreasing sequence of parameters such that $h^k_{\mathtt{max}} \to 0$

- Calibration of parameter ω^+: $\omega^+ = 0$

$$L^{\mathtt{select}} := \left\{ (x, \alpha) \in \mathcal{F}^k \mid h(x) = 0 \ \text{ and } \alpha = \alpha^k_{\max} \right\}$$

with $\alpha^k_{\max} = \max_{j=1,2,\ldots,|\mathcal{F}^k|} \alpha^j$

DMS-Filter Improvements

- If the poll center chosen is infeasible, we update the parameter $h^k_{\texttt{max}}$

$$h^{k+1}_{\max} := \begin{cases} \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^t\right) < h\left(x^k_I\right) \right\} & \text{if iter } k \text{ is improving,} \\ h\left(x^k_I\right) & \text{if } h\left(x^k_I\right) = \max\limits_{x \in I^k} h(x), \\ \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^k_I\right) \le h\left(x^t\right) < \max\limits_{x \in I^k} h(x) \right\} & \text{otherwise.} \end{cases}$$

  - Generates a nonincreasing sequence of parameters such that $h^k_{\texttt{max}} \to 0$
- Calibration of parameter ω^+: $\omega^+ = 0$

$$L^{\texttt{select}} := \left\{ (x, \alpha) \in \mathcal{F}^k \mid h(x) = 0 \ \text{ and } \alpha = \alpha^k_{\max} \right\}$$

with $\alpha^k_{\max} = \max_{j=1,2,\ldots,|\mathcal{F}^k|} \alpha^j$

DMS-Filter Improvements

- If the poll center chosen is infeasible, we update the parameter $h^k_{\mathtt{max}}$

$$h^{k+1}_{\max} := \begin{cases} \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^t\right) < h\left(x^k_I\right) \right\} & \text{if iter } k \text{ is improving,} \\ h\left(x^k_I\right) & \text{if } h\left(x^k_I\right) = \max\limits_{x \in I^k} h(x), \\ \max\limits_{x^t \in V^{k+1}} \left\{ h\left(x^t\right) : h\left(x^k_I\right) \le h\left(x^t\right) < \max\limits_{x \in I^k} h(x) \right\} & \text{otherwise.} \end{cases}$$

  - Generates a nonincreasing sequence of parameters such that $h^k_{\mathtt{max}} \to 0$

- Calibration of parameter ω^+: $\omega^+ = 0$

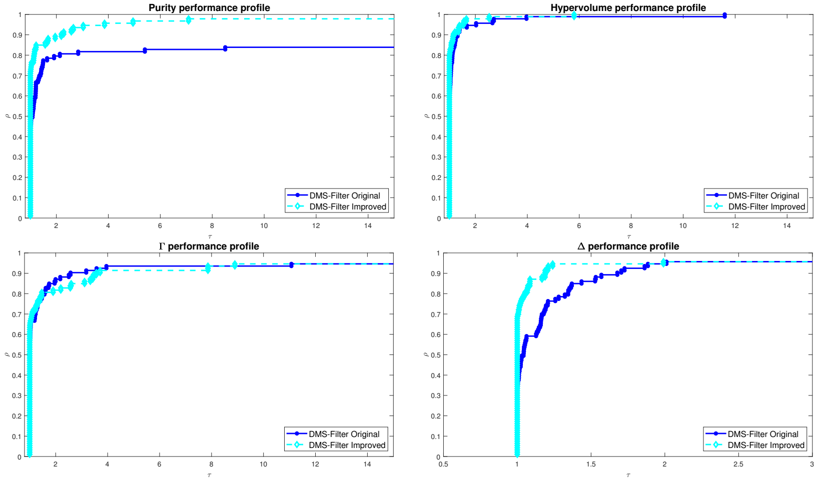
$$L^{\mathtt{select}} := \left\{ (x, \alpha) \in \mathcal{F}^k \mid h(x) = 0 \ \text{ and } \alpha = \alpha^k_{\max} \right\}$$

with $\alpha^k_{\max} = \max_{j=1,2,\ldots,|\mathcal{F}^k|} \alpha^j$

Numerical Settings

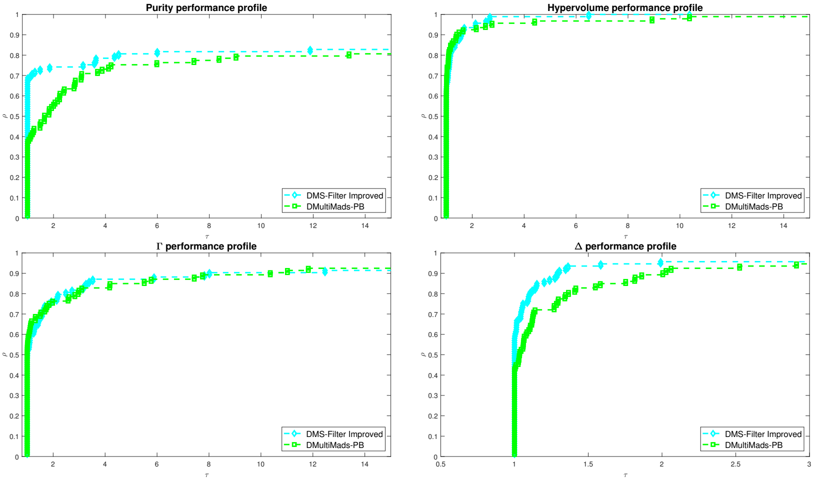
- Comparison between DMS-Filter Improved and DMultiMads-PB
- 93 biobjective problems with nonlinear constraints and bounds
  - number of variables between 3 and 30
  - number of constraints between 1 and 29
- Initialization in line
  - n-points equally spaced in the line segment, joining the variable upper and lower bounds
- Stopping criteria - one $\alpha_k < 10^{-9}$ and maximum of 20000 function evaluations

DMS-Filter (Original versus Improved)



DMS-Filter Original **DMS-Filter Improved**

DMS-Filter Improved versus DMultiMads-PB



DMS-Filter Improved **DMultiMads-PB**

Outline

- DMS-Filter extends filter methods to constrained Multiobjective Derivative-free Optimization
- DMS-Filter presents a well-supported convergence analysis for both globalization strategies
- DMS-Filter presents competitive numerical results for constrained Biobjective Derivative-free Optimization Problems

- Future work comprises extending the approach to problems with more than two objectives

THANKS FOR YOUR ATTENTION!

Any comments or questions?

Everton Jose da Silva - ejo.silva@campus.fct.unl.pt